# Throughput of Wireless Relay Networks via Minimum Evacuation Times and Index Coding.

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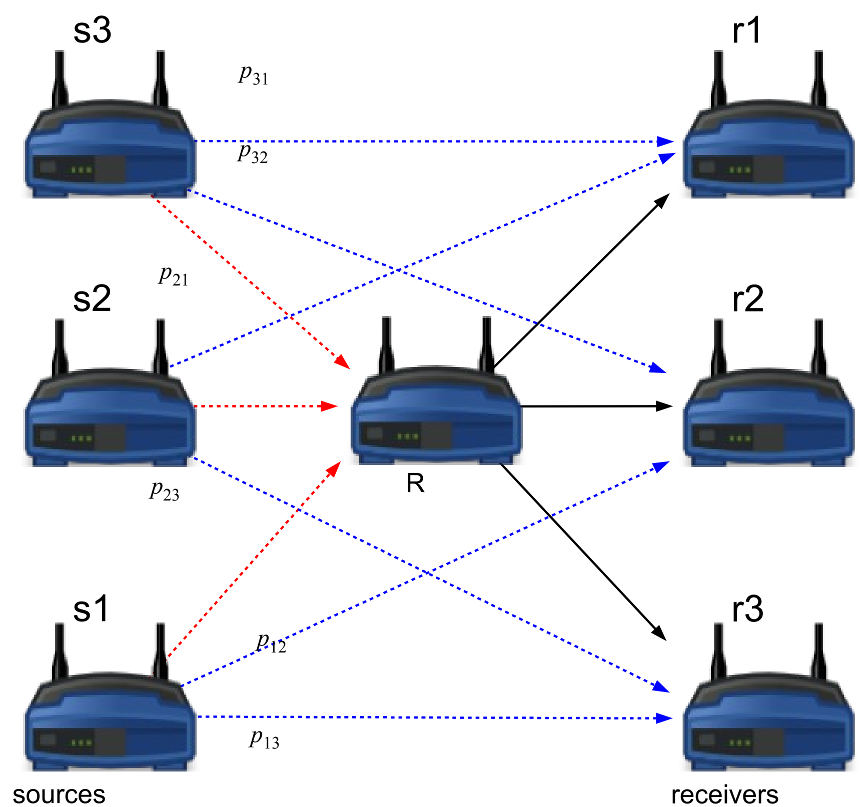
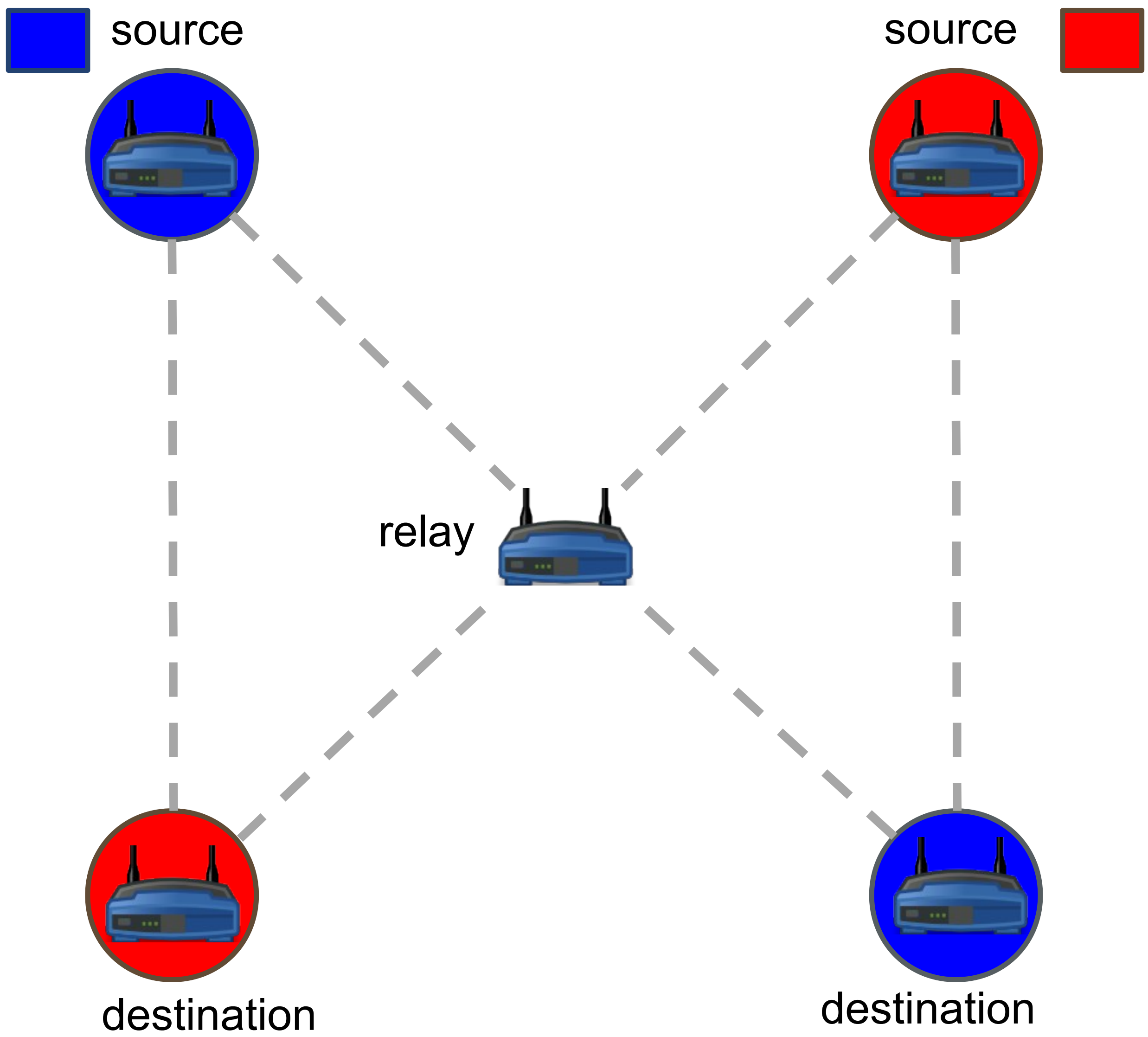
# Wireless Network Coding:

- N sources and receivers '1-1' paired in flows
- One hop network with relay between
- Probability that transmission of source i is overheard by receiver j≠i
- Maximize throughput



# Example:



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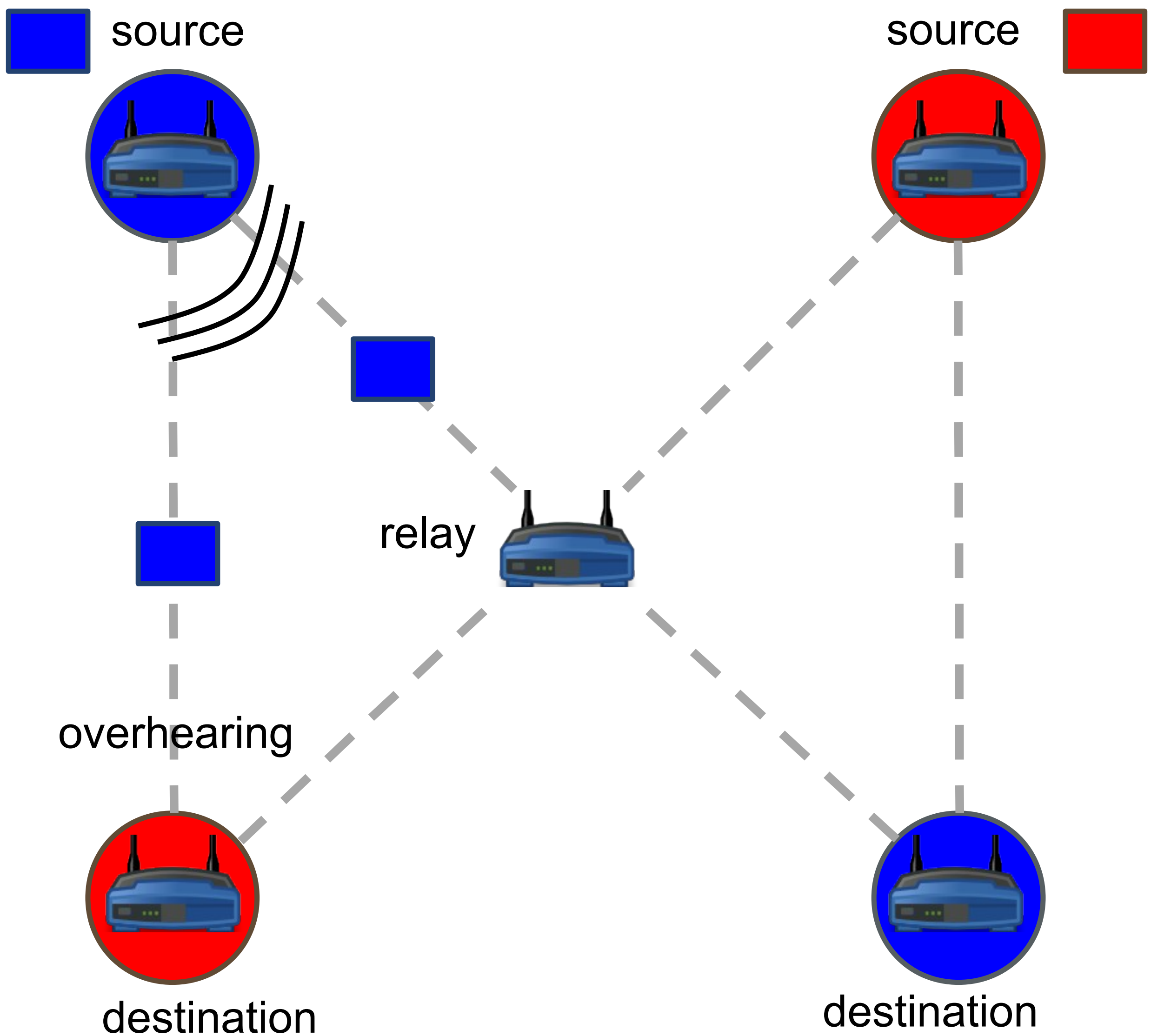
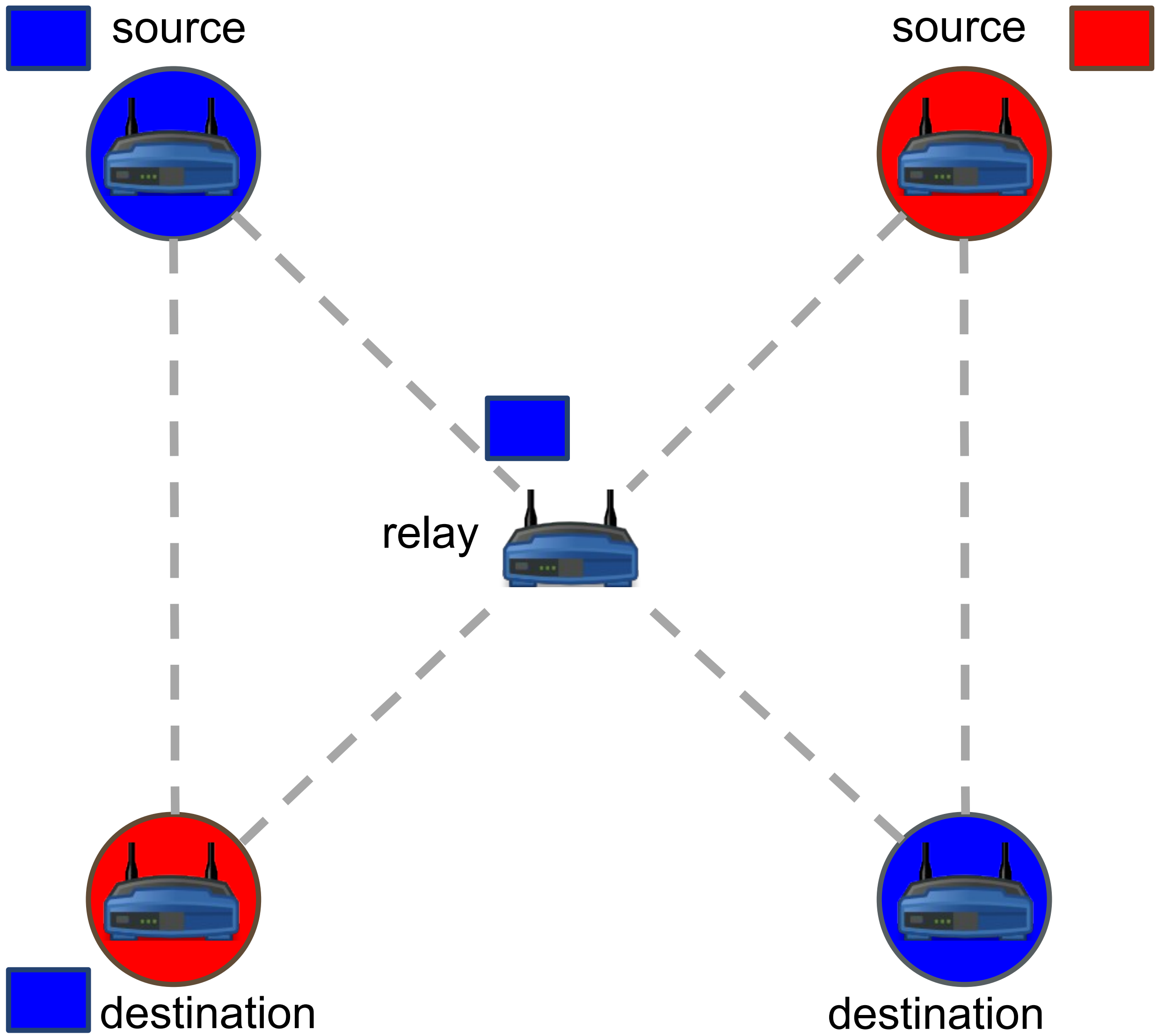

source
source
relay
overhearing
destination
destination

# Example:



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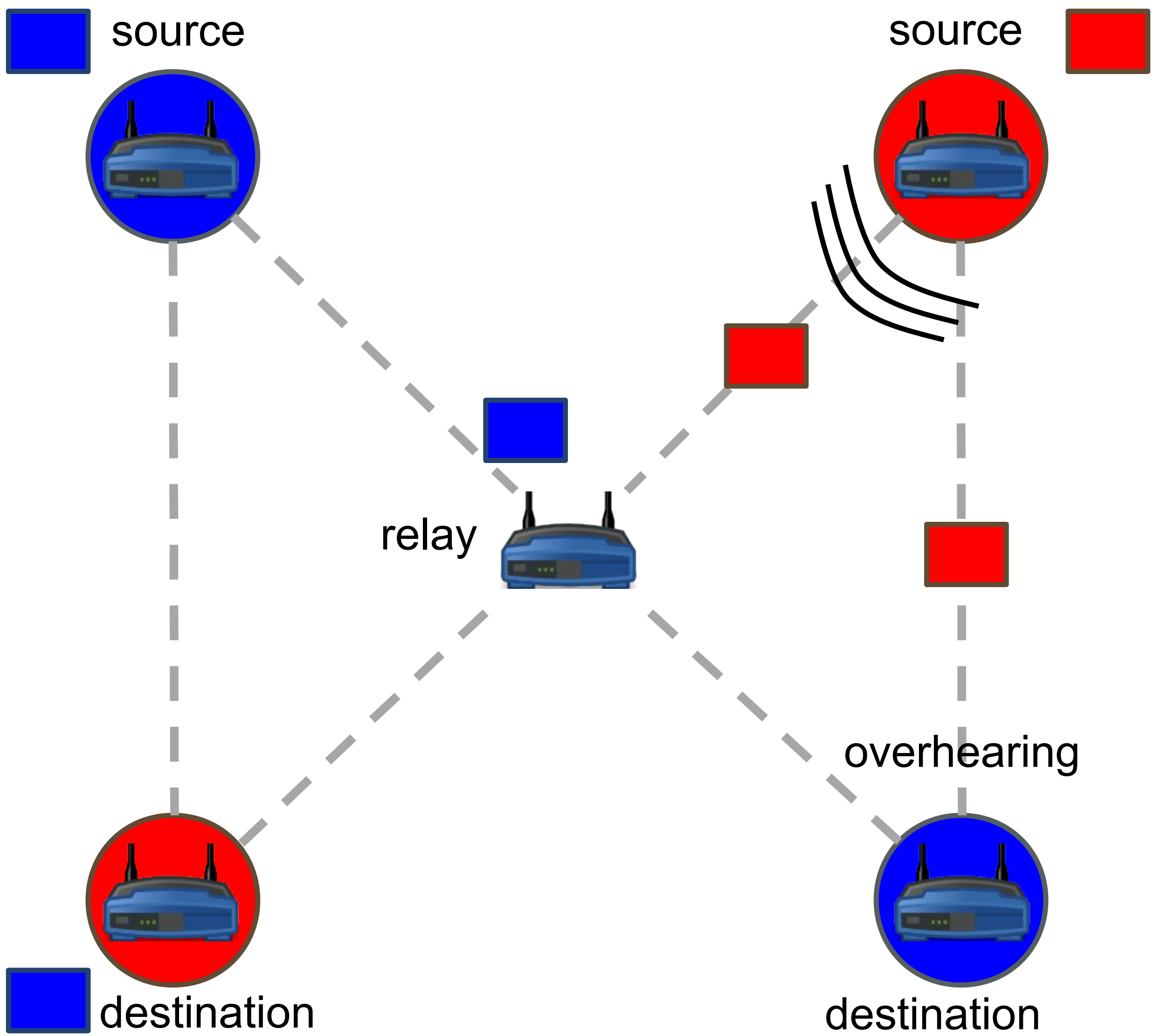

source
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relay
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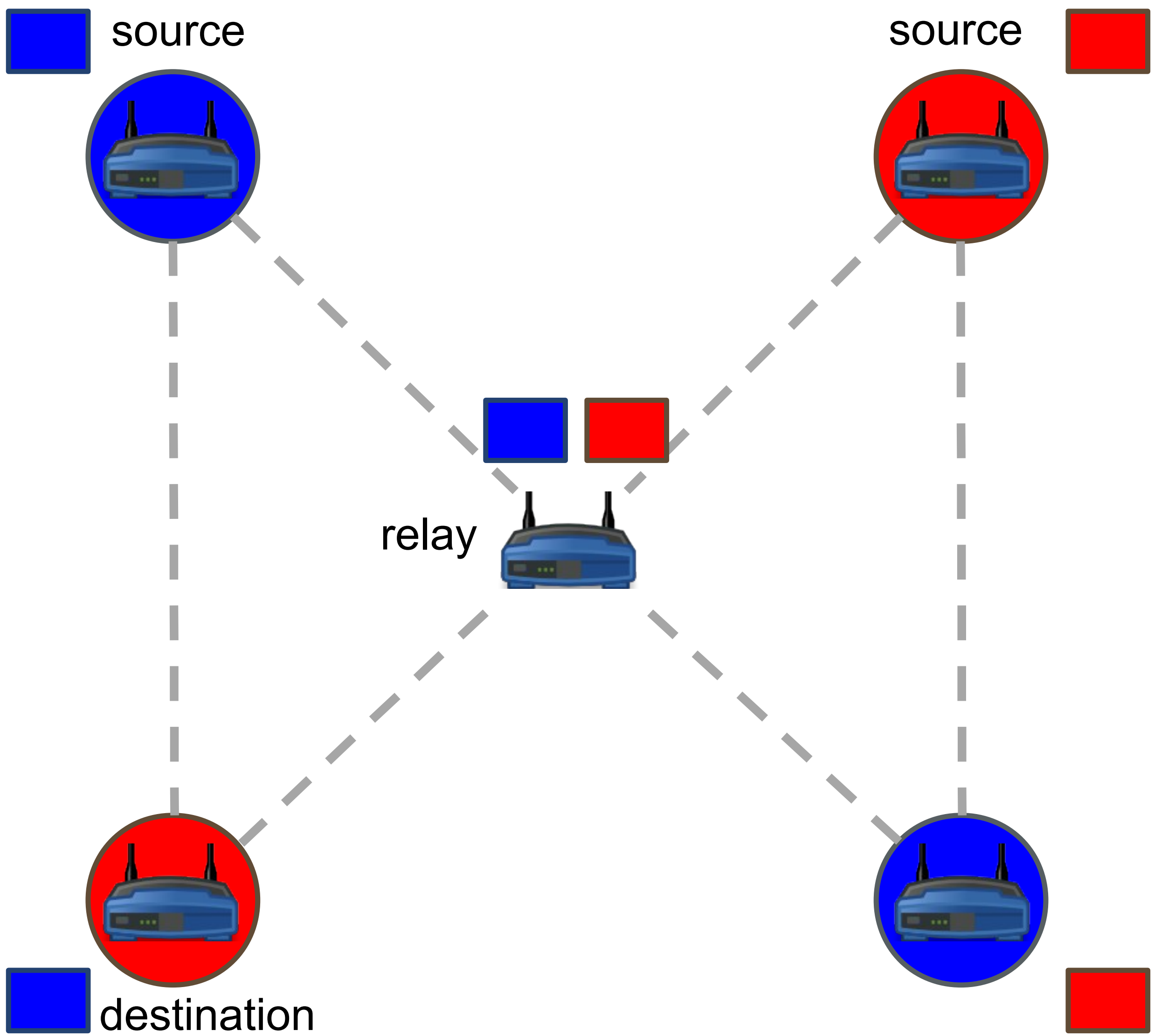
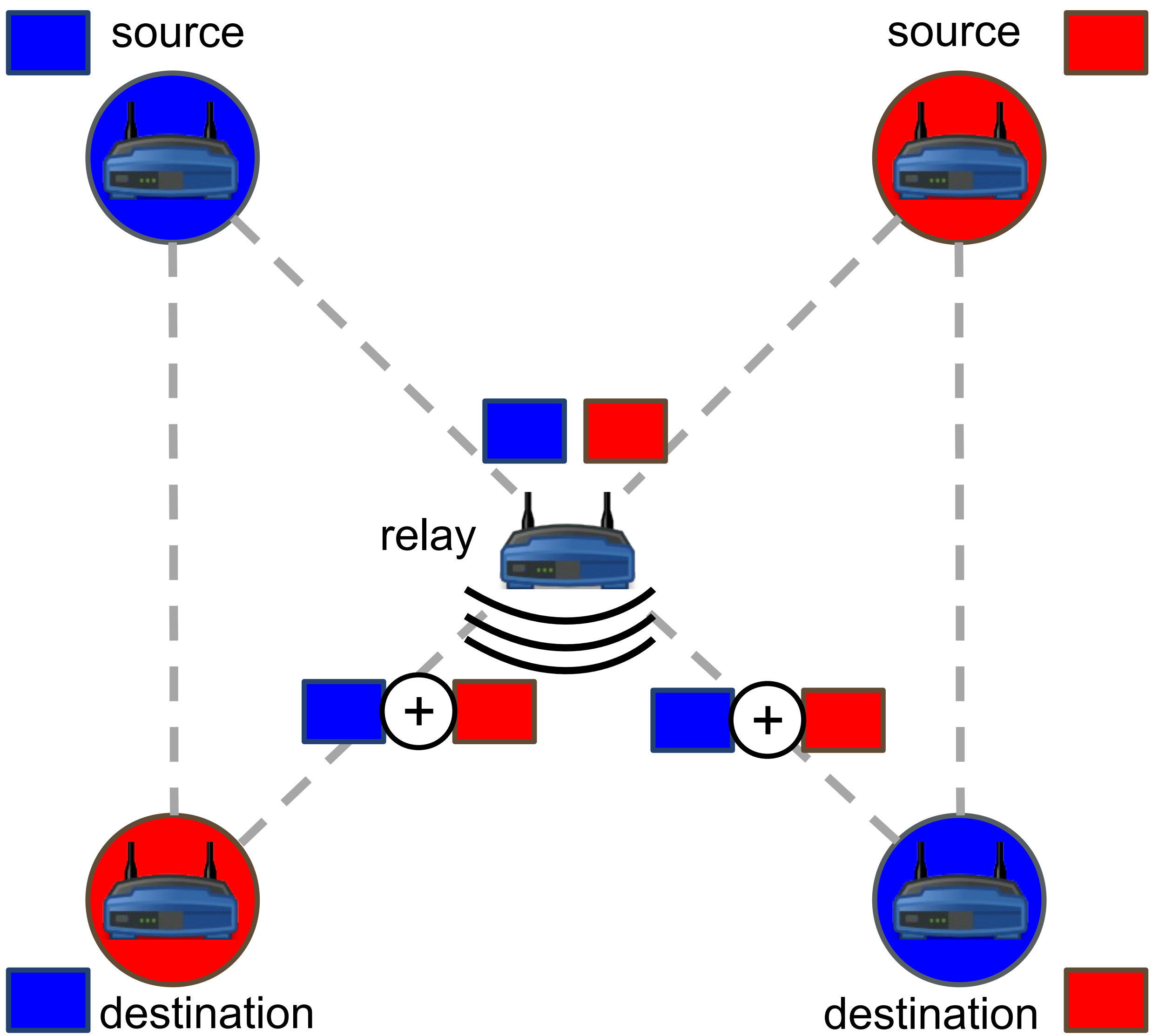


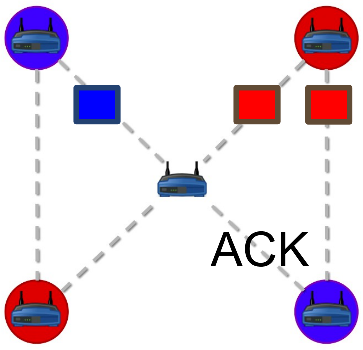
# Example:




- Both receivers decode in one slot instead of two.
- **But how does the relay know what packets were overheard?**

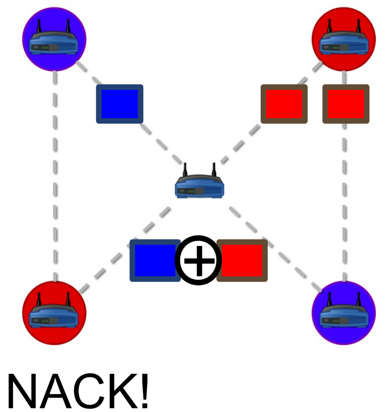
# Two models of feedback:

## ACK




- Nodes report each overhearing (send ACK)
- Relay learns which packets were overheard

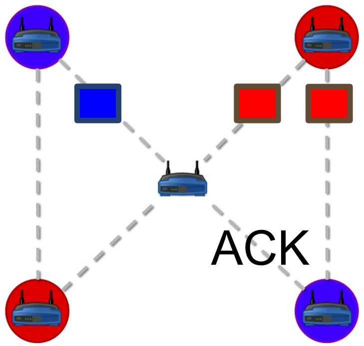
## NACK




- No ACKs
- Relay XORs blindly
- If there is a decoding failure the receiver sends NACK

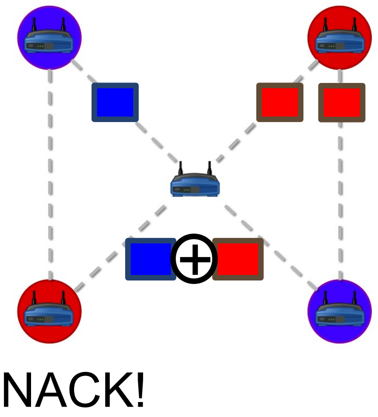
# Pros & Cons of feedback methods:

## ACK




No decoding failures
(Perfect knowledge of overhearing events)
**but**:
Large numbers of ACK reports

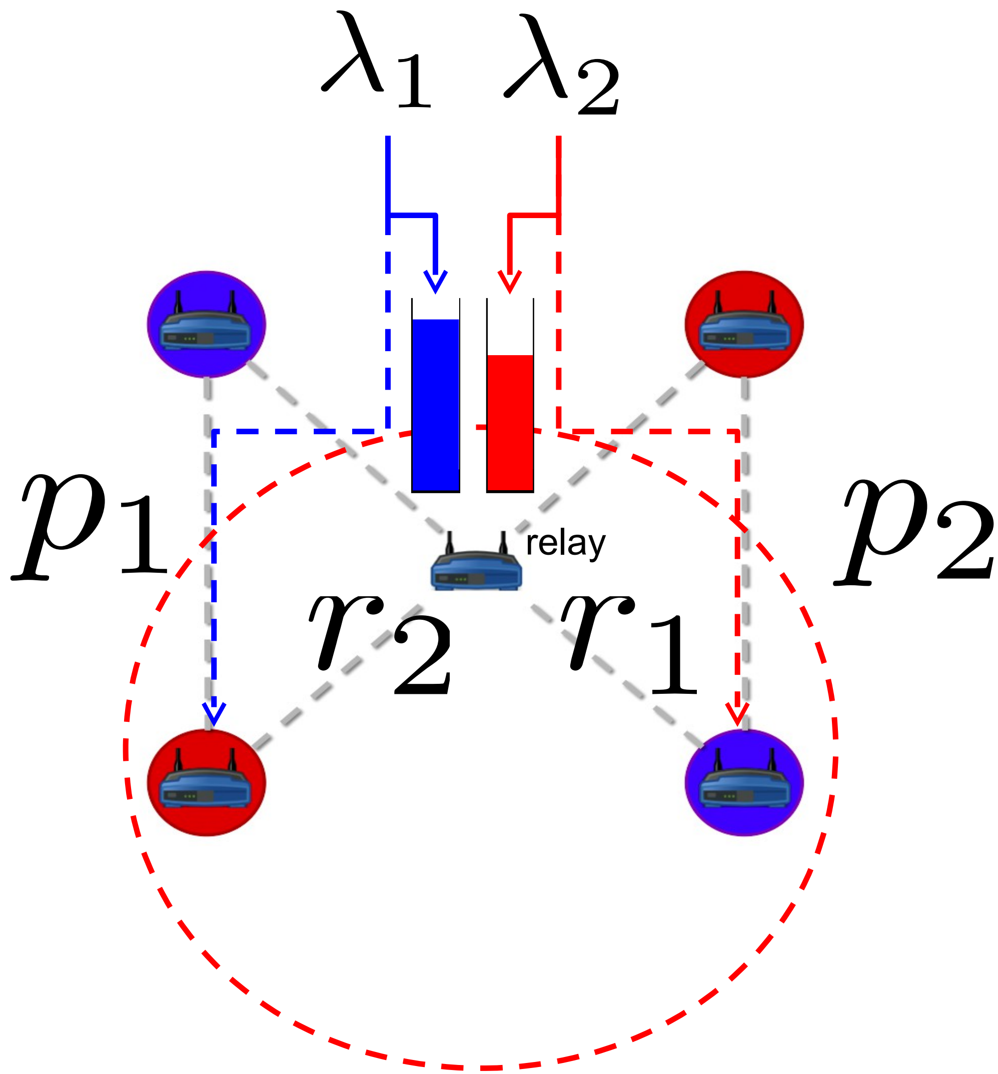
## NACK



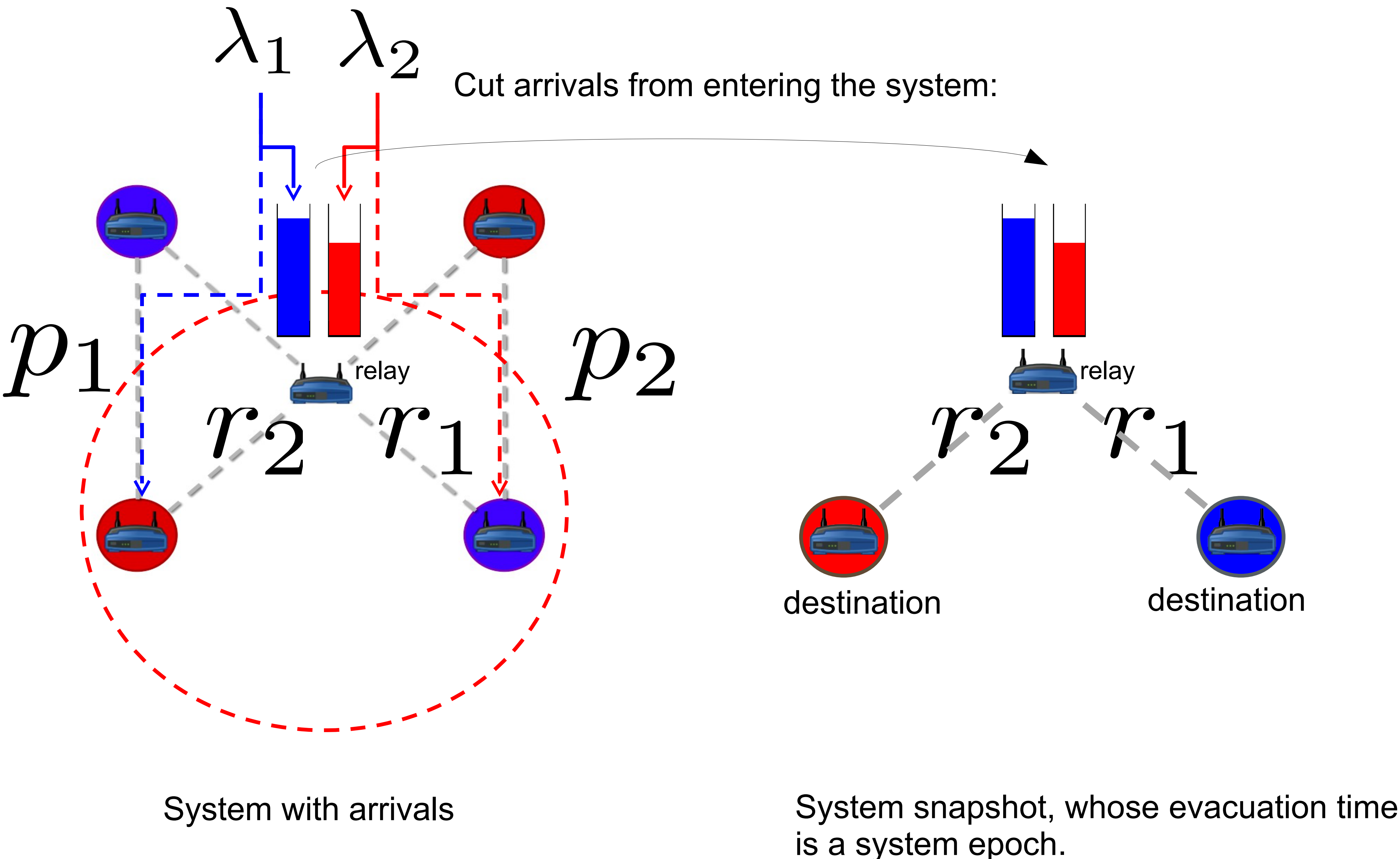

Smaller number of NACK reports
(depending on the algorithm used)
**but**:
Possible decoding failures
(Statistical knowledge of overhearing events)

# General WNC System model:

- Broadcast rates dictate reception.
- Probabilities of overhearing
- Control descision: pick an inter-flow XOR control and a rate.
- Relay queues hold packets to be sent.
- Receiver buffers hold packets that help decoding (including undecoded relay transmissions).



# System Snapshot & Epoch:



# Throughput Region via Evacuation Times:

Let $\overline{T}^{\star}(k)$ be the minimum average evacuation time of a system **snapshot** over all policies.

Then it holds that the throughput region of the system is the set of rates that satisfies:

$$\hat{T}(\boldsymbol{\lambda}) \leq 1.$$

where:

$$\hat{T}(\boldsymbol{\lambda}) = \lim_{t\to\infty} \frac{\overline{T}^{\star}(\lceil t\boldsymbol{\lambda} \rceil)}{t}.$$

and can be achieved by using the optimal evacuation policy at each epoch

L. Georgiadis, G. S. Paschos, L. Tassiulas, and L. Libman, “Stability and Capacity through Evacuation Times,” in Information TheoryWorkshop, (ITW), Sep. 2012.

# Contribution:

- 2-user ACK model:

  Throughput of our policy = Throughput region

- 2-user NACK model:

  Throughput of our policy

  =

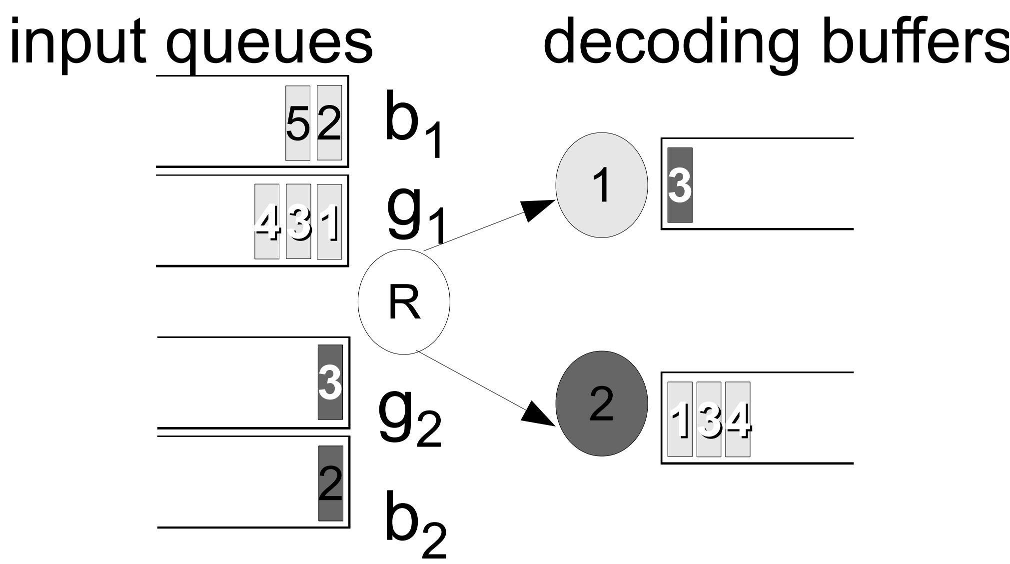
  Throughput region of XOR policies

  =

  Throughput region under equal rates

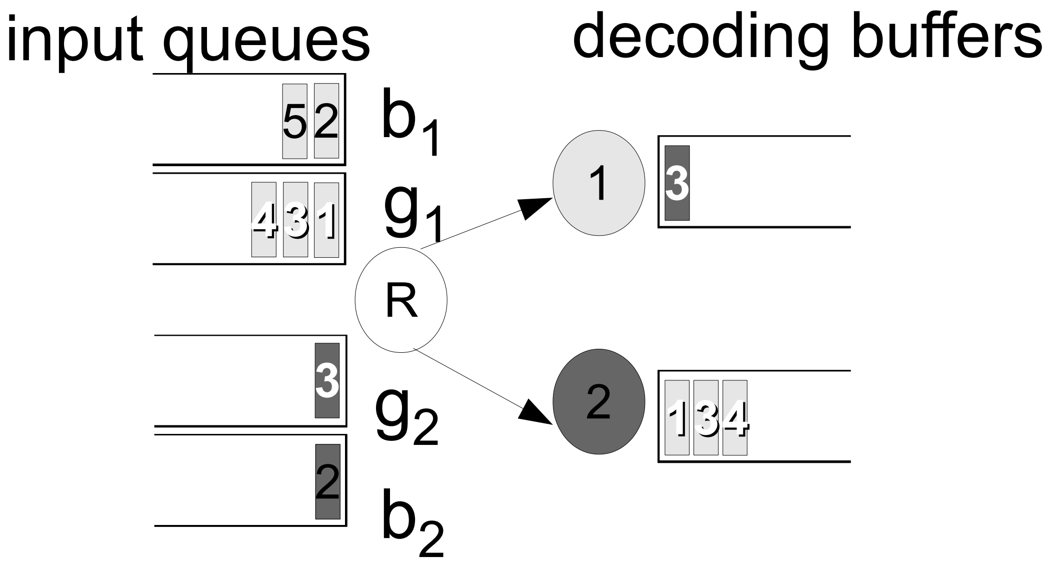
# 2-user ACK solution:

- Perfect overhearing information
- 2 queues for each flow, one for **good**(overheard) and one for **bad** packets.

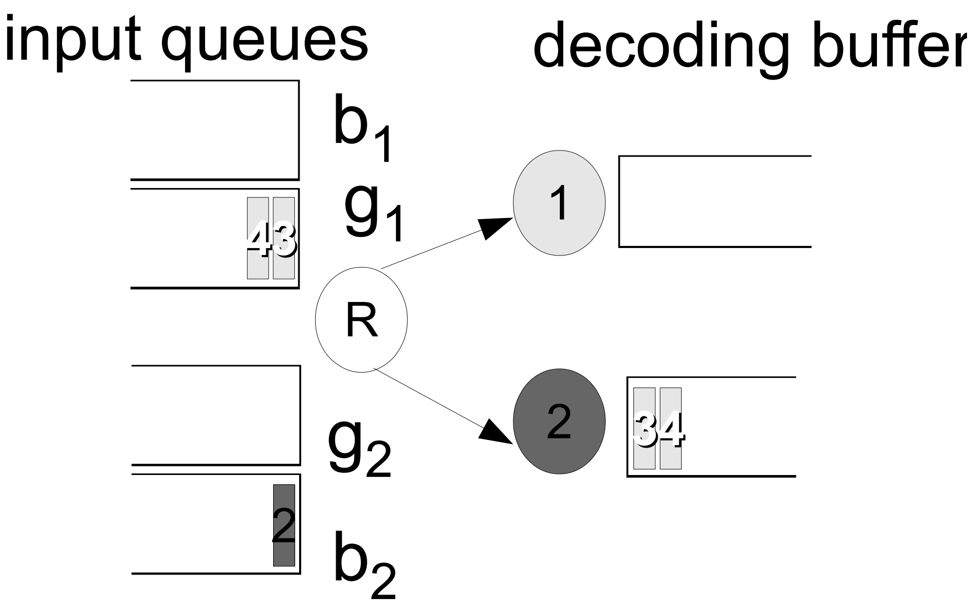



Initial system

# 2-user ACK solution:




Initial system




After controls g1 + g2 and b1 are used with r = 2

# 2-user ACK solution:

**Proposed Evacuation Policy:**

- Transmit g1+g2 until one type is not available (empty queue)
- Then use {g1,g2,b1,b2}
- Evacuation time:

$$T^{bdet}(k_1, k_2) \triangleq \frac{k_1}{r_1} + \frac{k_2}{r_2} - \frac{\mathbb{E}[\min\{N_1, N_2\}]}{\max\{r_1, r_2\}}$$

- Ni, i = 1,2 is the number of overheard packets of source i from receiver j ≠ i.

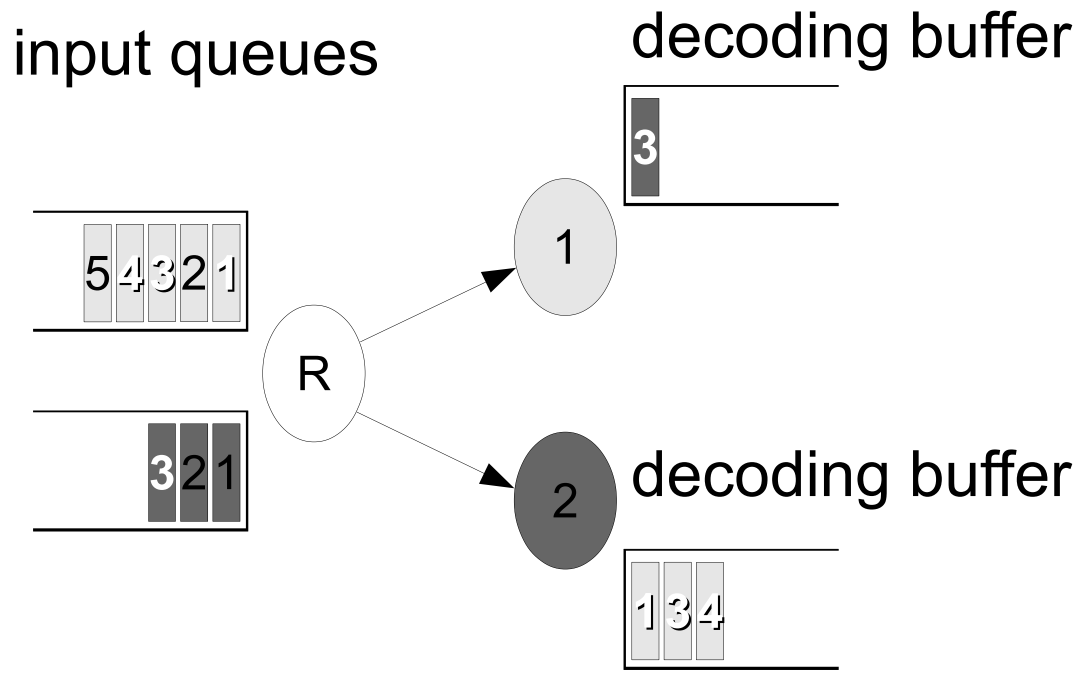
- Throughput region:

$$\frac{\lambda_1}{r_1} + \frac{\lambda_2}{r_2} - \frac{\min\{p_{12}\lambda_1, p_{12}\lambda_2\}}{\max\{r_1, r_2\}} \leq 1.$$

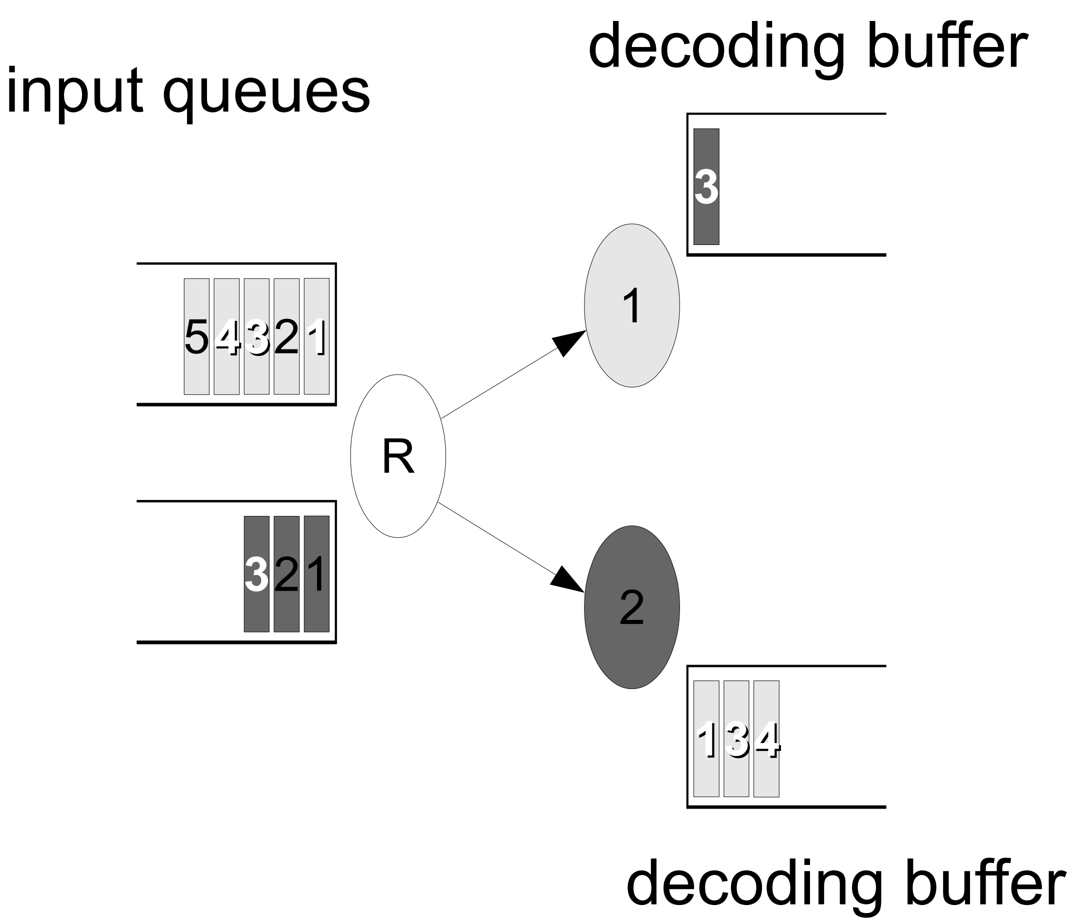
Can be proven to be optimal over arbitrary policies.

# 2-user NACK solution:

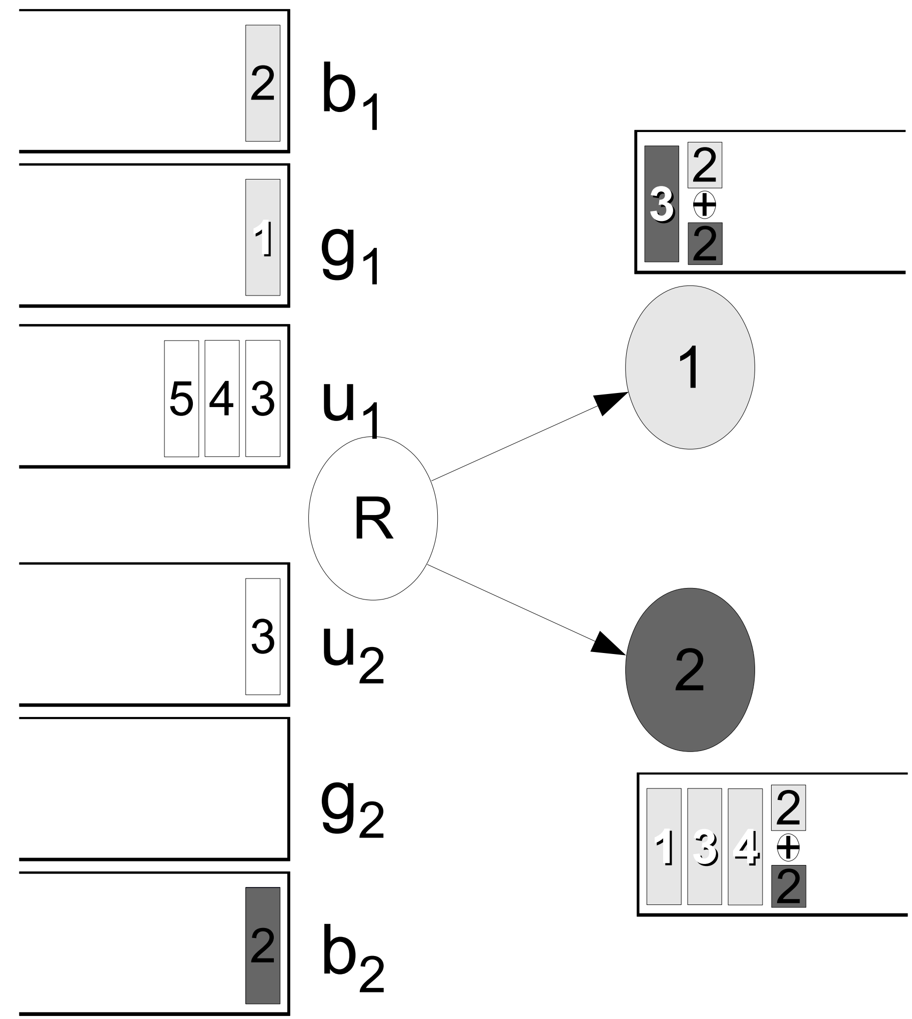
- Imperfect overhearing information
- 3 queues for each flow, one for **good**(overheard), one for **bad** and one for **unknown** packets (i.e no feedback for them yet.)
- Relay knows the overhearing probabilities.



# 2-user NACK solution:




Initial system:




After controls u1+u2 are used with r = 2

# 2-user NACK solution:

**Proposed Evacuation Policy:**

- If $1 - p_f > \frac{\min(r_1,r_2)}{\max(r_1,r_2)}$ send all packets singleton.
- Else:
  1) g1+g2
  2) u1+g2 or g1+u2
  3) u1+u2
  4) b1 or b2 or other singletons
- Code constrained (over any XOR policy) throughput region:

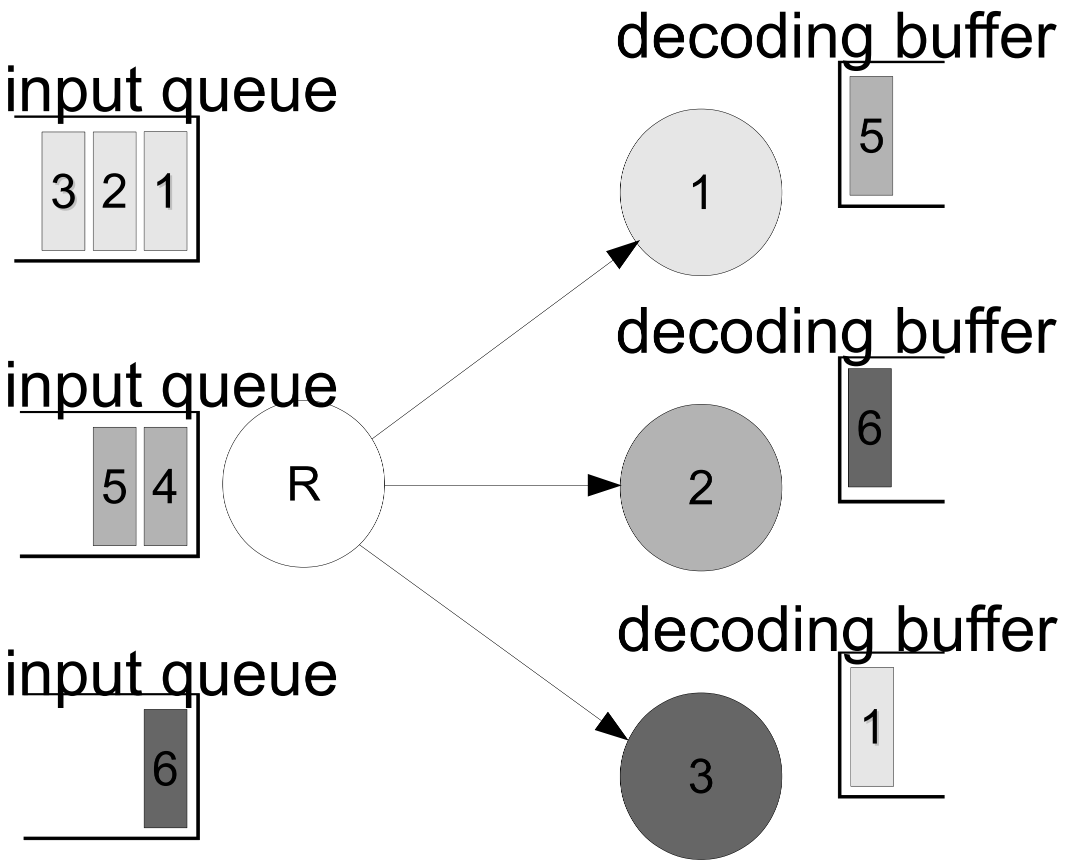
plugin $r_1 = r_2$ or $p_f = 1$
And get identical throughput region with the previous case!

$$\frac{\lambda_1}{r_1} + \frac{\lambda_2}{r_2} - \frac{\min\{\lambda_1 p_1, \lambda_2 p_2\}}{p_f}\left[\frac{1}{r_f} - \frac{1-p_f}{r_s}\right]^+ \leq 1,$$

# A related problem: Index Coding

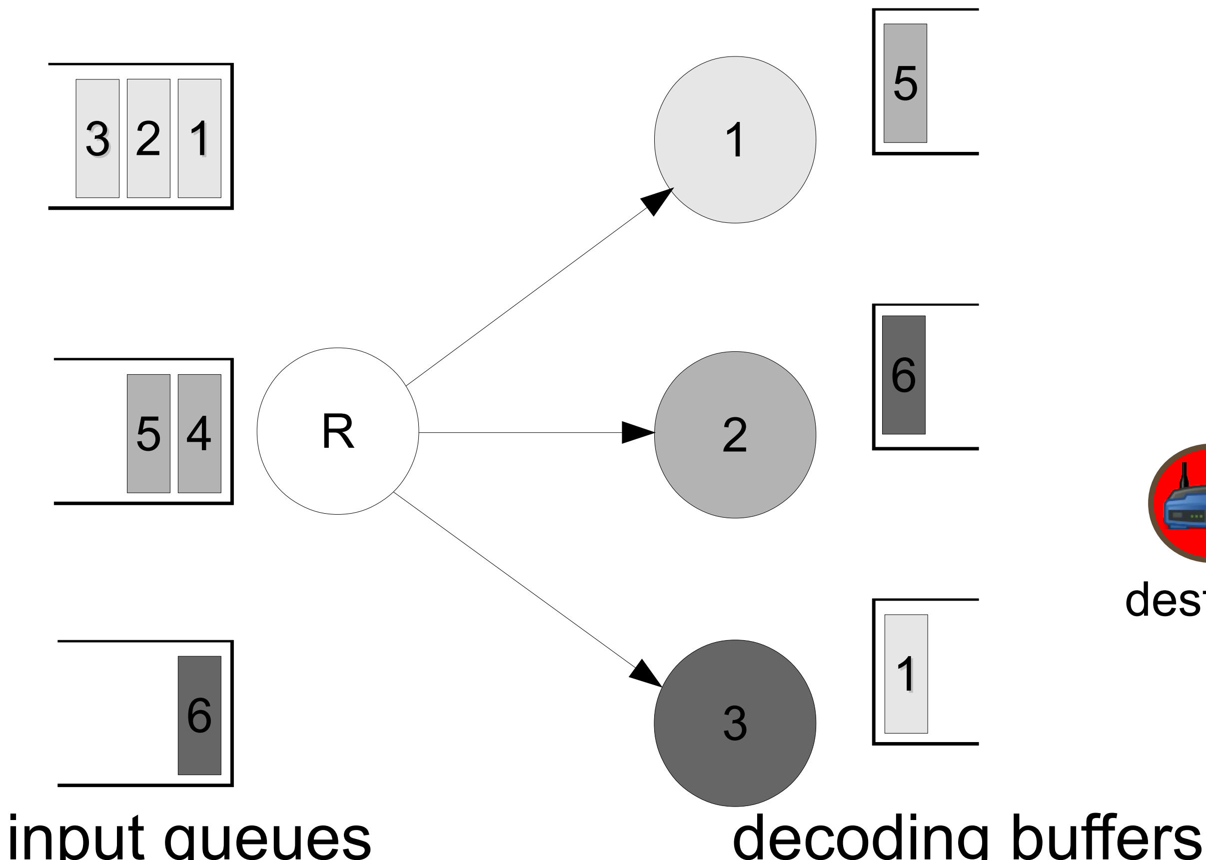
- N receivers and a relay
- Relay disseminates information to receivers with broadcasting.
- Packets destined to receiver i are known by receiver j≠i
- Broadcast channel

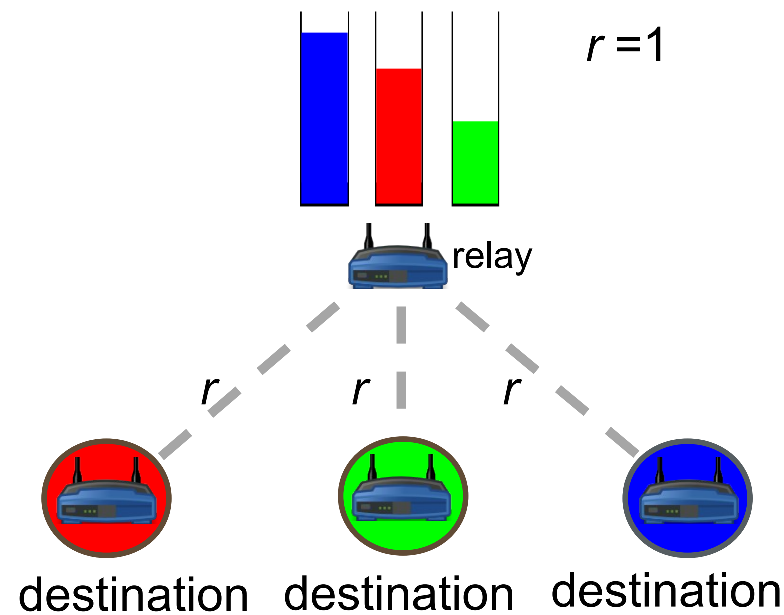
  (1 packet/slot)
- Maximize throughput



# Index Coding:

- Optimal Evacuation of a WNC snapshot can be reduced to an instance of Index Coding assuming the transmission rates are units and we have ACK feedback!

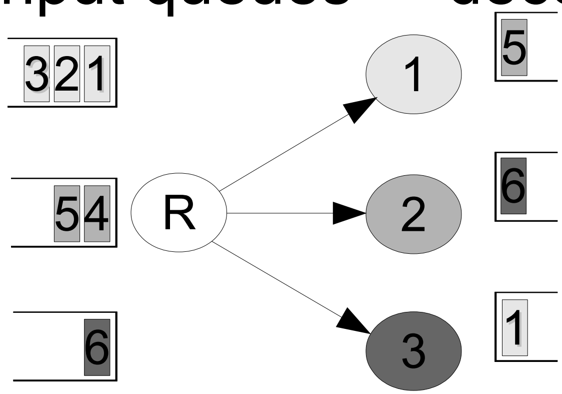


# Contribution:

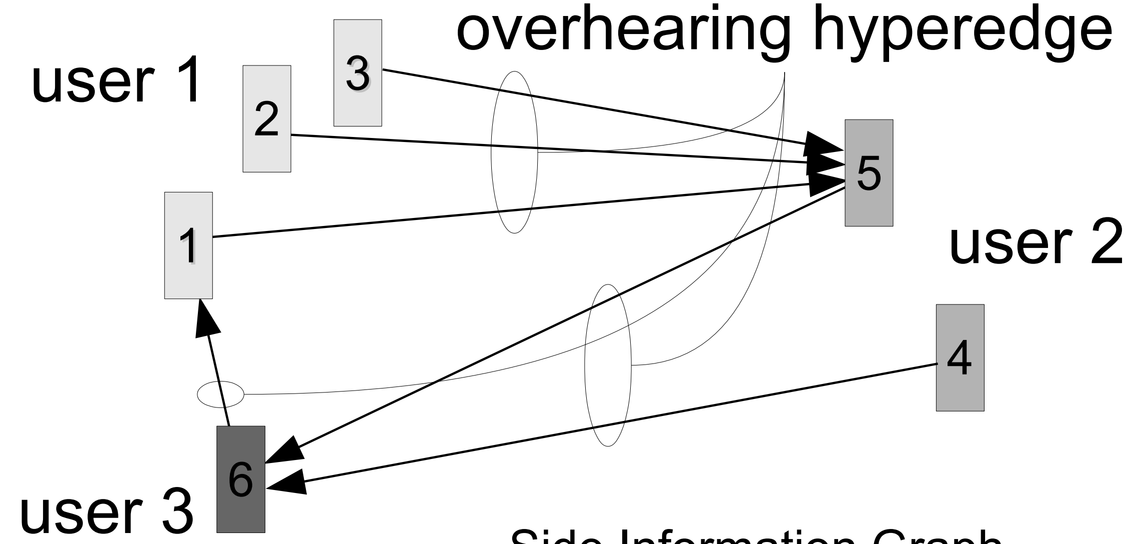
- Index Coding:
  - We offer a problem decomposition
  - We can achieve minimum evacuation time with XOR codes and no intra-flow XOR coding
- Multi-user ACK model:
  - We offer a Heuristic

# Index Coding (previous results):




Initial Problem

Abstracts to:




Side Information Graph

Is fitted by:

rec. 1 rec. 2 rec. 3

$$\mathcal{A} = \left[\begin{array}{ccc|cc|c} 1 & 0 & 0 & 0 & * & 0 \\ 0 & 1 & 0 & 0 & * & 0 \\ 0 & 0 & 1 & 0 & * & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * \\ \hline * & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Matrices that fit the SIG

Whose minimum rank achieves:

$$\min_{C \in \mathcal{I}_G^L} len(C) = minrk_2(G)$$

For this example, minimum rank = 5

# Index Coding (previous results):

$$\mathcal{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

One of the solutions

Pick maximal linear independent set of rows:

$$\mathcal{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Corresponding packets: 1 2 3 4 5 6

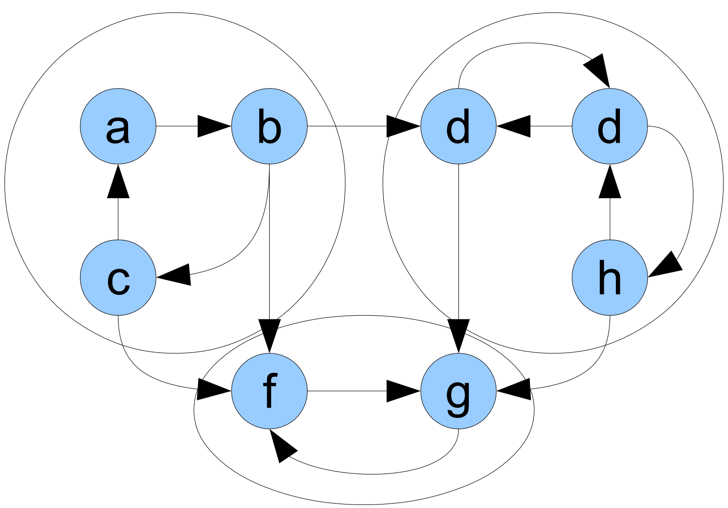
Which results in transmitting:

1 + 5
2
3
4
1 + 6

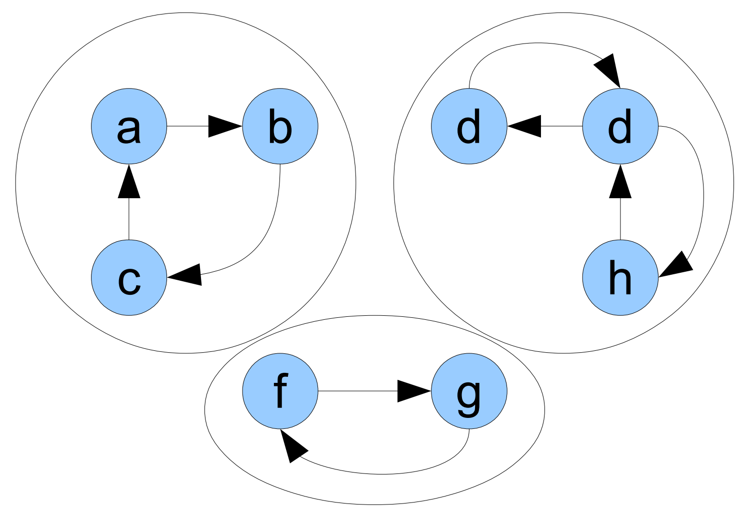
Z.B. Yossef, Y. Birk, T. S. Jayram, and T. Kol, "Index Coding With Side Information,"

# Index Coding (our results):

- The minimum rank of any SIG is the sum of the minimum ranks of its strongly connected components.




Minimum rank of this graph...




...is the minimum ranks of these graphs!

# Index Coding (our results):

- There is no loss of optimality if we constrain our coding options to omit intra-flow coding.

$$A = \left(\begin{array}{ccc|cc|c|c}
1 & 0 & 0 & * & * & * & * \\
0 & 1 & 0 & * & * & * & * \\
0 & 0 & 1 & * & * & * & * \\
\hline
* & * & 0 & 1 & 0 & * & * \\
* & * & 0 & 0 & 1 & * & * \\
\hline
* & 0 & 0 & * & 0 & 1 & * \\
\hline
* & 0 & 0 & * & 0 & * & 1
\end{array}\right)$$

Problem

$$S = \left(\begin{array}{ccc|cc|c|c}
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\hline
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
\hline
1 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right)$$

Solution with no intra-flow coding

# Multi-user ACK Heuristic

- We use the above result and omit intra-flow coding.
- Transmit codes with the greatest packets/timeslots ratio (solutions derived from minimum rank).
- If there are ties, code the packets that are least overheard.
- All other ties are solved arbitrarily
- We expand on multiple rates by demanding that a control is chosen only when the corresponding queues have at least the number of packets suited for this rate.

# Rival Policies (IDNC):

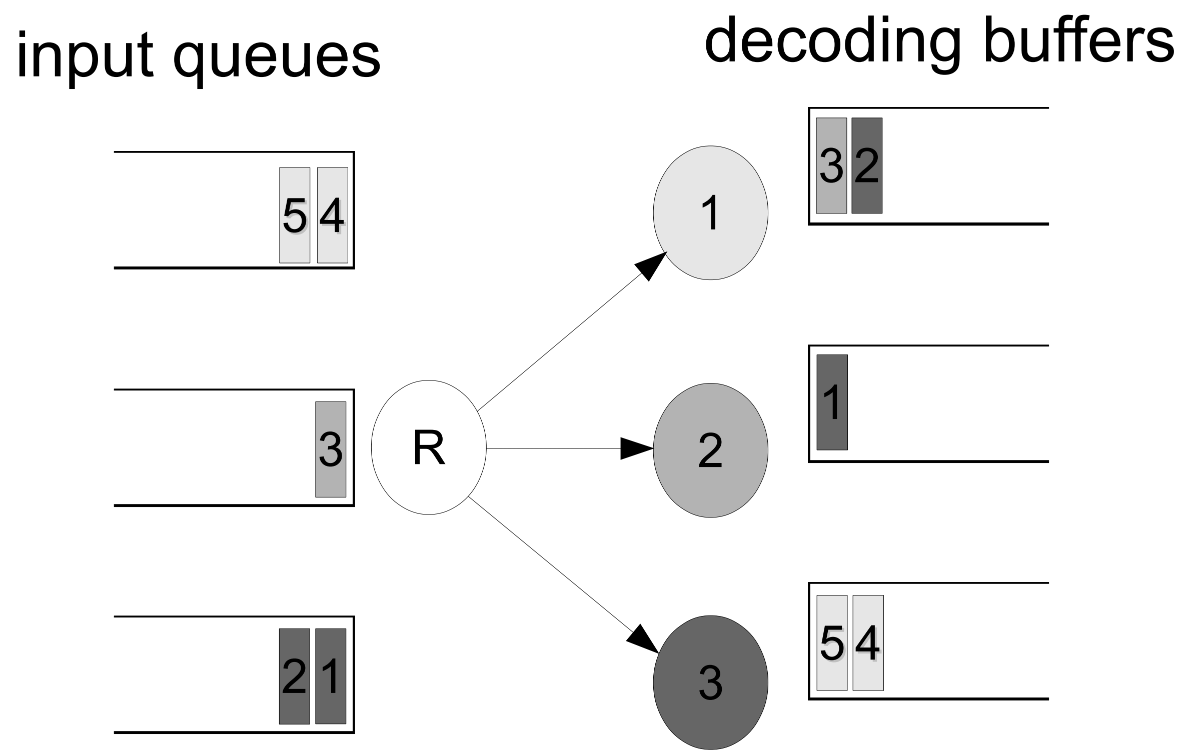
- Immediately Decodable Network Codes
- Choose the greatest available inter-flow code that can be decoded immediately (i.e based only on the original side information)
- Solve ties arbitrarily

# Rival Policies (RLNC):

- Random Linear Network Coding
- M packets to send
- N = minimum number of overheard packets over all receivers
- Randomly generate M-N linearly independent codewords and transmit them.
- Randomness increases the probability of linear independence with the number of equations
- All receivers decode all packets

# Rival Policies:

- Let $r_1 = r_2 = 1$
- How will Heuristic fare vs RLNC and IDNC with this system?
- Controls 1+3, 3+4 are not ID !
- RLNC sends 4 LCs !



# Simulation Results (2-user):

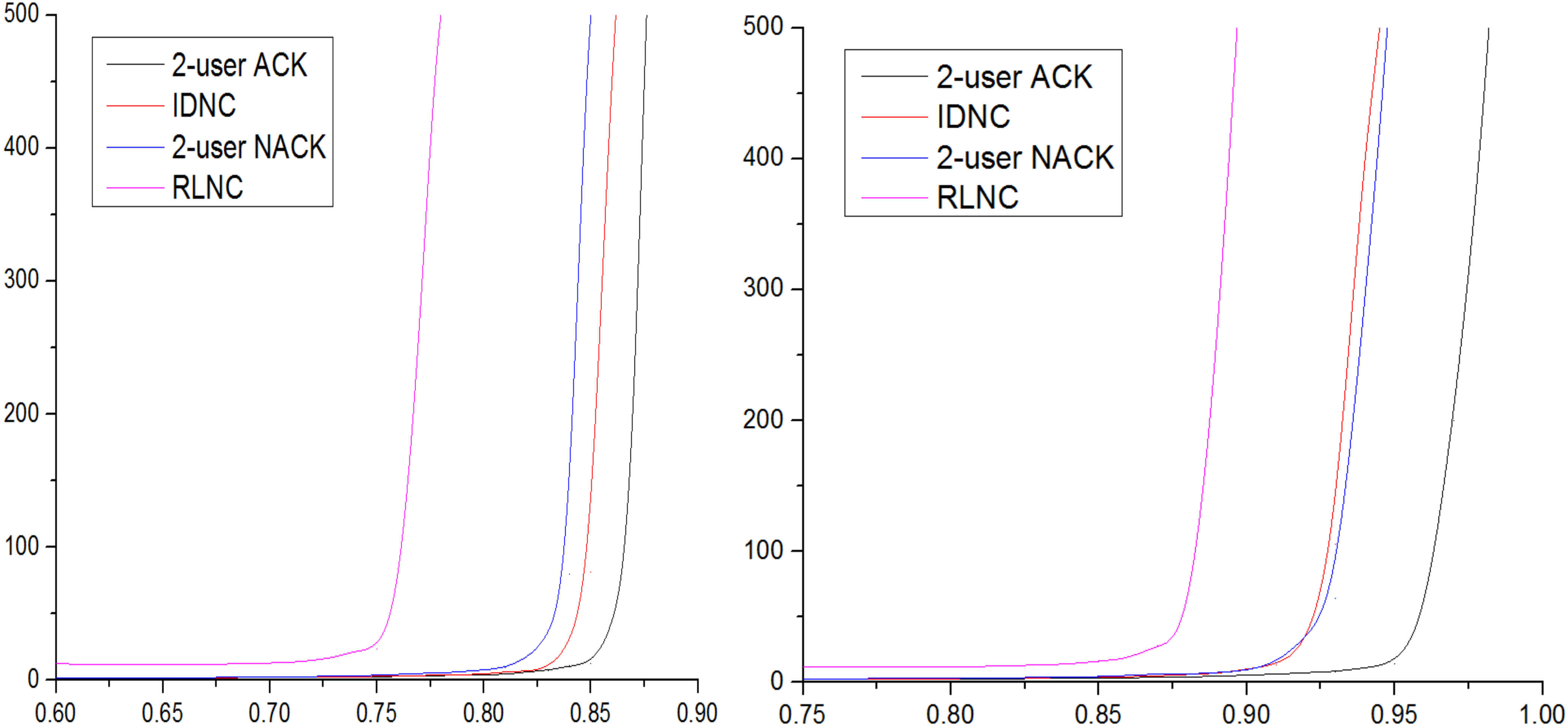



Figure 8: Average delay performance for a system with 2 receivers, in two chosen directions $\boldsymbol{\lambda} = (\lambda, \lambda)$(left) and $\boldsymbol{\lambda} = (\lambda, .8\lambda)$ (right). The overhearing probabilities are: $p_1 = .9, p_2 = .7$. For both cases, the rates are $r_1 = 2, r_2 = 1$

# Simulation Results (Multi-user):

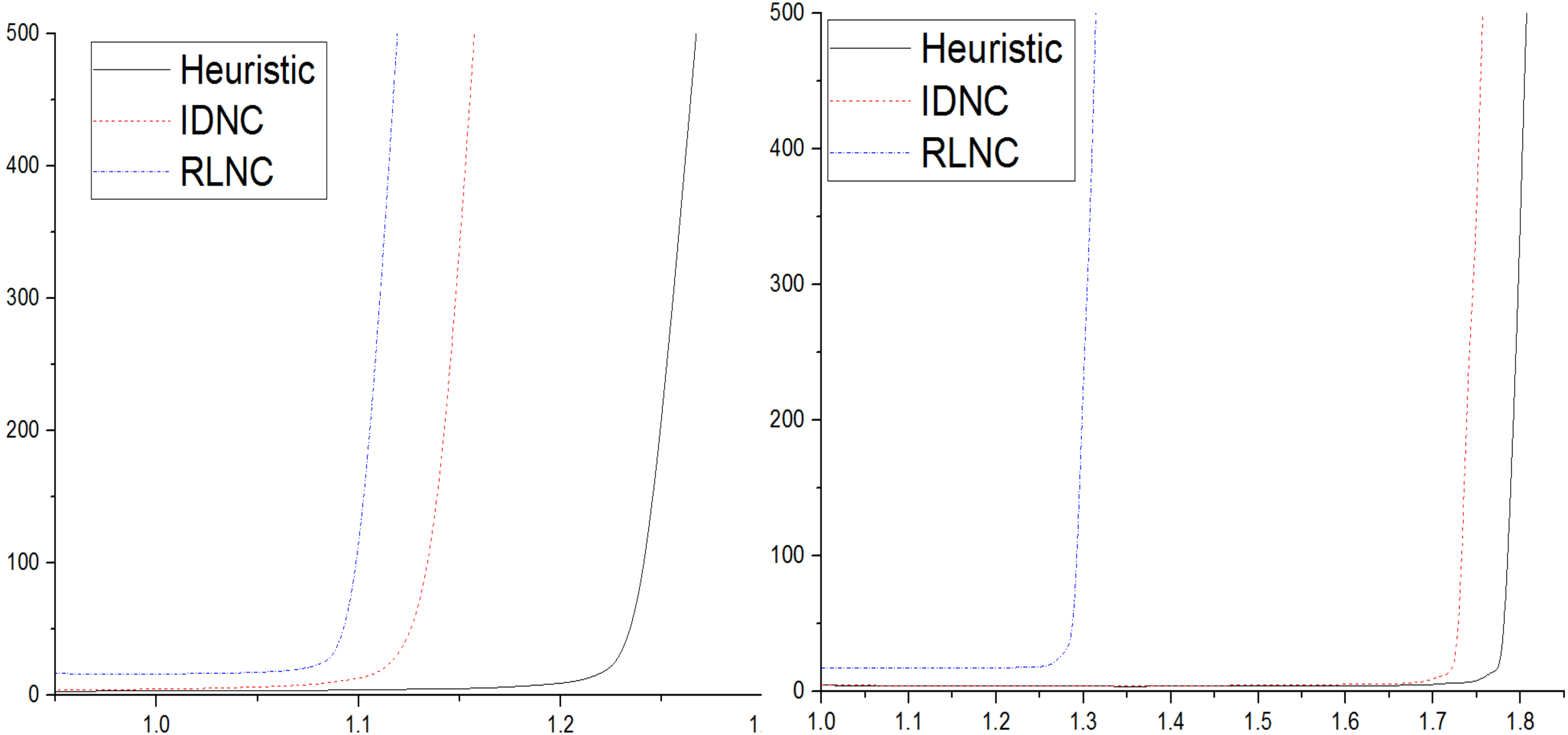



Figure 9: Average delay performance for a system with 2 receivers, in two chosen directions $\boldsymbol{\lambda} = (\lambda, \lambda, \lambda)$ , $r_1 = 2, r_2 = 2, r_3 = 2$ (left) and $\boldsymbol{\lambda} = (\lambda, .8\lambda, .6\lambda)$ , $r_1 = 4, r_2 = 3, r_3 = 2$(right). The overhearing probabilities are: $p_{12} = p_{23} = p_{31} = .8$, $p_{13} = p_{21} = p_{32} = .5$.

# Future Avenues:

- Expand 2-user NACK and ACK cases for arbitrary number of users.
- Possibly with the help of Index Coding.

Questions?