A Comparison Performance Analysis of QoS WLANs. Approaches with Enhanced Features

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Abstract— **The main contribution of this work is to compare and enchance already known methods for performance analysis of the IEEE 802.11e MAC layer, such as the use of Markov chains, queueung theory and probabilistic analyses. The output MAC layer metrics of our work are thoughput, total delay and accurate values of queuing and MAC delay using moments of the z-transform. The PDF of the MAC delay is also given as input for the queueing service time. The comparison metrics are based on complexity, flexibility and accuracy. In addition our analyses incorporate a gaussian error-prone channel in 802.11e and results are provided taking into account the Block-ACK, which is a key feture of the forthcoming IEEE 802.11n. The correctness of results are validated via Opnet Modeler.**

I. Introduction

The widely deployment of WLANs has set pace to extensive scientific studies of the IEEE 802.11 standard [1]. In addition heterogeneous multimedia applications require advanced editing over the standard, so as to accomplish specific QoS characteristics [2]. The core 802.11e standard propose a new Hydrid Coordination Function (HCF), which has a HCF Controlled Channel Access (HCCA) and an Enhanced Distributed Coordination Access (EDCA), mechanism which are capable of offering access according to specific QoS features..

A typical literature search demonstrates that there exist a number of performance analyses for the 802.11e [3], [4], [5], [6], [7], [8], [9],which incorporate Discrete Time Markov Chains (DTMC). Other models, such as [12], [10] and [11] use alternative methods of analysis. A scientific tendency has exploited since [3] first presented his case study, to enchance analyses, and provide more accurate performance values. However the subjercts maturatity, leaves small space for new analyses that could prompt scientific interest. Our avant-garde approach is to propose amendments over these known analytical methods, find their acccurate values and open a new field of performance comparison. It is straightforward that since new protocols tend to be analysed by either DTMCs, queueing theory, or general probabilistic methods; the results of the proposed methods can be used to find the best method of analyzing forthcoming or known standards.

We have used three known models [6], [10] and [11], which depict the three known methods of analysis, and they are extended according to QoS features proposed in the dot11e standard and error-prone channel. The first model is a DTMC analysis,which takes into account the state of the previous slot, into the Markov Chain. Whereas, the second one, is based on elementary conditional probability arguments, and finally in the third one queueing theory and Little's Theorem were used to analyze the standard. Accurate values of delay and a way for calculating the PDF of the MAC delay is also given while compared to a Metamodeling technique. The proposed analyses carry by themselves scientific interest, and could be studied separately. The models are corrected and extended to calculate the MAC, the queueing and the total delay.

Except from the already well investigated features of the dot11e, in our models the effect of different retransmission limits among the access categories is implemented. Other characteristics of the standard such as the Freezing of backoff counters is taken into account. A more accurate equation of saturation throughput is provided and a way of incorporating the intercollision phenomenon among the Access Categories. The dot11e standard in order to accomplish specific QoS requirements, lets delay-prone multimedia applications to have higher transmission probability. This is accomplished with increased backoff times and other features. However it was noted that higher ACs monopolized relatively quick the channel, especially when the type of multimedia traffic was bursty. After D4.0 of the IEEE 802.11e standard [2], the standard defined that after a succesful transmission the AC should get to backoff, and to contest again for the channel. This means that the state 0 of the Markov chain can not be chosen after a successful transmission, a feature that older models have omitted or have partly mentioned it [13] and [14].

In fact the proposed models include also gaussian erroneous channel for EDCA, analyzing in the same environment the effect of Block-ACK and the efficiency of the new IEEE 802.11n introducing much higher transmission rates. Most of the previous analyses [15],[20] implement the transmission failure probability in the solution of the Markov Chain, which is not correct since the Markov Chain does differentiate collision and errors, but are implemented in the Performance Analysis. In our analysis this corrected while adding new features and giving the exact solution with RTS/CTS and basic access mode. The simulation results are based on the HCCA model included in the last version of Opnet Modeler^{TM} 12.

The proposed models require advanced knowledge of [2],

[3] and [10], since formulas and other proved explanations are taken as prerequisites. The paper is organised as follows. In Section 2 we provide numerical analysis of the transmission probability (τ_i) and Mean Backoff Duration ($E[BD]_i$) of each of the three models. In Section 3, using the above values of each model, throughput and delay values (MAC, queueing, total delay and the accurate pdf of these) using various transmission rates, conditions of the channel and features enabled or disabled. A third subsection is also given for the analysis of the Block-ACK feature. In section 4 validation, results and comparison is provided. In the last Section a conclussive discussion is made upon advantage and disadvantage of each one.

II. Numerical Analysis of the Models

In [2] Quality of Service is succeeded via using four Access Categories (AC) with different transmission parameters each. The standard uses different values of $AIFS[i]$, $CW\min_i$, $CW\max_i$ and backoff persistent factor (πf_i). Another feature, Transmission Opprtunity (TxOP) which is the maximum amount of bytes that a station is allowed to transmit consecutively, before it releases the channel. However here all ACs send packets with equal number of bytes below TxOP limit. We define as $W_{i,0} = CW_{i,\min} + 1$. where for $W_{i,j}$ is the contention window size and j is the backoff stage. m_i is defined as the retry limit, after which the contention window stays the same for a number of retransmissions. When the backoff exponential algorithm reaches L_i times of retransmission and there is a collision the packet is dropped. In the legacy 802.11 [1], persistent factor (πf_i) has the value of 2, which means that after every collision the backoff contention window doubles its value. In [2] persistent factor can have different values according to each Access Category.

$$W_{i,j} = \begin{cases} \lfloor (\pi f_i)^j W_{i,0} \rceil & j = 0, 1, ..., m_i \\ \lfloor (\pi f_i)^{m_i} W_{i,0} \rceil & j = m_i + 1, ..., L_i \end{cases} \tag{1}$$

where $\lfloor \cdot \rceil$ is the closest integer function.

Before defining the mathematical analysis, the following assumptions have been made regarding all models. The number of station N_i is finite,the same for each AC and contend only in a single-hop network and there is a constant packet generator so as the network to be saturated, which means that there is always a packet ready to transmit. The channel is erroneous, with uniform distributed errors, and there are no hidden terminal, capture effects and link-adaptation mechanisms.

A. Markov Chain Model

A four dimension DTMC is proposed which presents the effect of contending terminals on the channel access probability of each Access Class (AC), and is decribed by the stationary probabilityies $b_{i,w,j,k}$. $i = \{0, 1, 2, 3\}$ describes the four Access Categories, which differentiate the access method according to the dot11e standard [2].

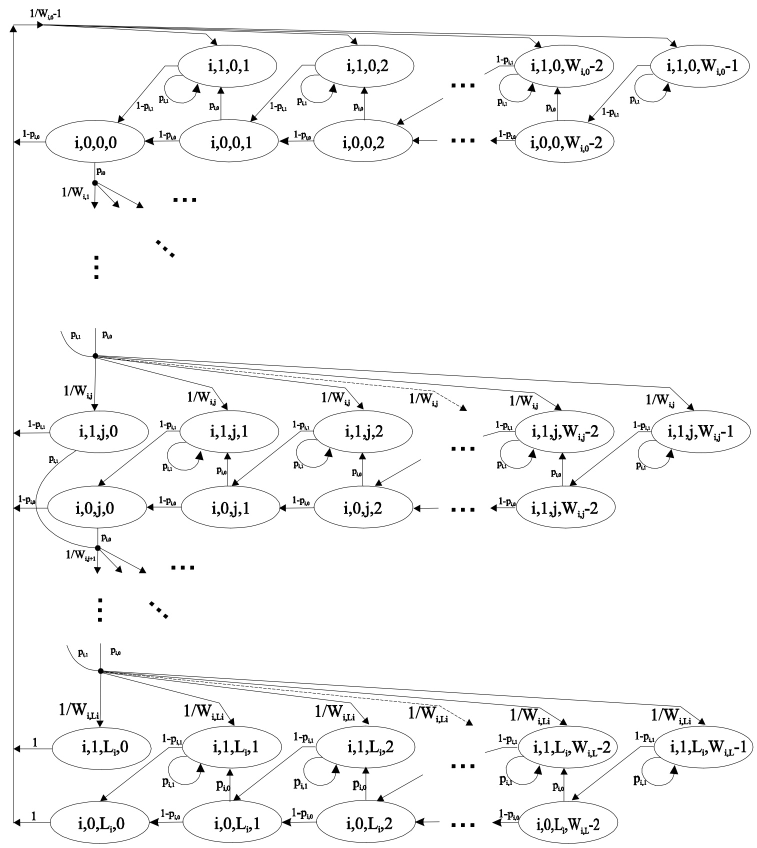

Fig. 1. Analytical Markov Chain for each Access Category AC[i].

The second dimension, w represents the condtion of the previous slot, where 1 is for the busy channel and 0 for the idle channel. Similarly to [6] a division is needed since special cases exist according to the state of the previous slot. If it was idle, all Access Categories of all station may access the channel if their backoff counter is decremented to zero. On the other hand if the previous slot was busy, another division must take place. Busy slot can occur if there is a collision or a transmission of another station. In the first case the stations that did not participate in the collision have frozen their backoff counter and will not be able to transmit. Whereas, the stations that collided can transmit in the next slot if they choose a new backoff value equal to 0. In the second case, when there is a successful transmission, none of the stations can transmit in the next time slot. This happens specifically for the standard IEEE 802.11e [2] and not for the legacy IEEE 802.11 [1]. The latest defines that after a successful transmission the contention window starts from 1 and not 0. All these are considered in the provided analysis and are shown in the markov chain of Fig. 1, which refers to each Access Category seperataly. Note that the state {i,1,0,0} is missing.

The other two symbols are, j for the backoff stage decribed above and k accounts for the backoff delay and takes values $k \in [0, 1, ..., W_{i,j} - 2]$ for $w = 0$, $k \in [\ 0, 1, ..., W_{i,j} - 1]$ for $j > 0$ and $w = 1$, and $k \in [1, ..., W_{i,j} - 1]$ for $j = 0$ and

$w = 1$. In [6] a similar Markov Chain is used for the legacy dot11. This model is extended considerably so as to include all the new characteristics of dot11e and a finite retry limit. [6] also solves the problem according to a state that in our modeling does not exist.

The probabilitiy $p_{i,0}$ (or $p_{i,1}$) that another terminal's Access Category is tranmsitting after an idle period (or after a busy period), without errors. The opposite one, that the channel remains idle after an idle period is represented q_0 (or after a busy period q_1). After these explanations all the transitions of the Markov chain has been verified and the following equations are accrued.

$$b_{i,1,j,0} = \psi_{i,j} b_{i,0,0,0} \tag{2}$$

for $j = 1, 2, ..., Li$

$$b_{i,1,j,k} = \frac{1 + p_{i,0}\left(W_{i,j} - 1 - k\right)}{1 - p_{i,1}} \psi_{i,j} b_{i,0,0,0} \tag{3}$$

for $k = 1, 2, ..., W_{i,j} - 1$ and $j = 0, ..., Li$

$$b_{i,0,j,k} = \left(W_{i,j} - 1 - k\right) \psi_{i,j} b_{i,0,0,0} \tag{4}$$

for $k = 0, 1, ..., W_{i,j} - 2$ and $j = 1, ..., Li$
where

$$\psi_{i,j} = \begin{cases} \dfrac{1}{W_{i0} - 1} & j = 0 \\ \dfrac{p_{i,0}}{W_{i,1}} & j = 1 \\ \dfrac{p_{i,0}}{W_{i,1}} \Pi_{i,j} & j = 2, 3, ..., m_i \\ \dfrac{p_{i,0}}{W_{i,1}} \Pi_{i,m_i} P_{i,j} & j = m_i + 1, ..., L_i \end{cases} \tag{5}$$

$\Pi_{i,j}$ and $P_{i,j}$ are defined

$$\Pi_{i,j} = \prod_{x=2}^{j} \left[\frac{p_{i,1}}{W_{i,x}} + \frac{p_{i,0}}{W_{i,x}} (W_{i,x-1} - 1) \right]$$

$$P_{i,j} = \prod_{x=m_i+1}^{j} \left[\frac{p_{i,1}}{W_{i,m_i}} + \frac{p_{i,0}}{W_{i,m_i}} (W_{i,m_i} - 1) \right]$$

Applying the normalization condition for each Access Category's Markov Chain, as each exponential backoff algorithm runs independently, we have

$$\sum_{k=0}^{W_{i,0}-2} b_{i,0,0,k} + \sum_{k=1}^{W_{i,0}-1} b_{i,1,0,k} + \sum_{j=1}^{L_i} \left[\sum_{k=0}^{W_{i,j}-2} b_{i,0,j,k} + \sum_{k=0}^{W_{i,j}-1} b_{i,1,j,k} \right] = 1 \tag{6}$$

after solving this equation $b_{i,0,0,0}$ is found

$$b_{i,0,0,0} = \frac{2(1 - p_{i,1})}{K_i + \Lambda_i} \tag{7}$$

$$K_i = W_{i,0}(1 - p_{i,1}) + p_{i,0}(W_{i,1} - 1)(2 - p_{i,1}) + \\ +2p_{i,0}(W_{i,0} - 2) + 4$$

$$\Lambda_i = \sum_{j=2}^{L_i} \psi_{i,j} W_{i,j} \left[\left(W_{i,j} - 1\right)\left(1 - p_{i,1} + p_{i,0}\right) + 2 \right]$$

The probabilities of accessing the channel in a time slot, whether the previous slot was idle or busy, are given by the following equations

$$\tau_{i,w} = \begin{cases} \dfrac{\sum_{j=0}^{m_i} b_{i,0,j,0} + \sum_{j=m_i}^{L_i} b_{i,0,j,0}}{P_{idle}} & w = idle \\ \dfrac{\sum_{j=1}^{m_i} b_{i,1,j,0} + \sum_{j=m_i}^{L_i} b_{i,1,j,0}}{1 - P_{idle}} & w = busy \end{cases} \tag{8}$$

where P_{idle}, is derived by the solution of $P_{idle} = q_0 P_{idle} + q_1(1 - P_{idle})$, and describes the probability that the channel is idle in the previous time slot (take notice that this different from the current idle slot symblozed below as P_{idle}).

1) Successful Transmission Probability: The probabilities that the channel remains idle after an idle (or a busy) time slot, can be straighfarward found by supposing that no other station transmits in that time slot.

$$q_w = \prod_{i=0}^{3} \left(1 - \tau_{i,w}\right)^{N_i}$$

The probability of another AC tranmitting is relatively complex. Except from the other station's ACs transmitting, an intercollision handler and virtual collision handler must also be taken into account. In the proposed analysis such a collision handler is also implemented, adding also a correlation measure which gives a close approximation of the intercollision problem. The phenomenon of intercollision happens when two ACs have different AIFS, and the one with the higher AIFS and higher $E[\Psi]$ has a smaller backoff value. Thus it may happen that these ACs will collide and the differentiation offered from the use of AIFS will be lost. See Fig.2.

$$r(i_1, i_2) = \max[1 - \frac{AIFS[i_1] - AIFS[i_2]}{E[\Psi]}, 0], i_1 \geq i_2 \tag{9}$$

Where $E[\Psi]$ is the mean consecutive number of idle slots. min is used to maintain the accurancy of the model.

$$E\left[\Psi\right] = \min(\frac{P_{idle}}{1 - P_{idle}}, 1) \tag{10}$$

This specific correlation measure simplifies the analysis, because it does not increase the complexity of the mathematical analysis when trying to solve the Markov Chain. The probabilities of a transmission failure (taking into account the

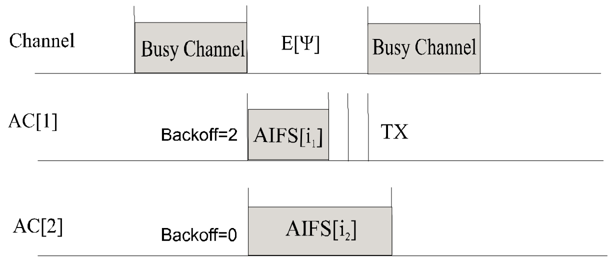

Fig. 2. The phenomenon of intercollision in two ACs of the same station

collision probability and error probabilities) after an idle or busy slot thus are

$$\begin{aligned} p_{i,0} &= 1 - \prod_{z<i} \left(1-\tau_{z,idle}\right)^{round[N_z \cdot r(z,i)]} \times \\ & \times \left(1-\tau_{i,idle}\right)^{N_i-1} \prod_{z>i}^{3} \left(1-\tau_{z,idle}\right)^{N_z} \end{aligned} \tag{11}$$

$$p_{i,1} = 1 - \left(1-\tau_{i,busy}\right)^{N_i-1} \prod_{z>i}^{3} \left(1-\tau_{z,busy}\right)^{N_z}$$

Since the errors are uniformaly distributed, the error events are indepently distributed (iid), thus the frame error probability is given by

$$p_{Se,i} = 1 - \left(1-p_{e,i}^{data}\right)\left(1-p_{e,i}^{ACK}\right)\left(1-p_{e}^{RTS}\right)\left(1-p_{e}^{CTS}\right)$$

$p_{e,i}^{data}$ and $p_{e,i}^{ACK}$ which show the uniformly distributed errors in the data packet and in the acknowledgement, and the same for the the probabilities $p_{e,i}^{RTS}$ and $p_{e,i}^{CTS}$ which are used only in RTS and CTS access method. If Basic access method is used then $p_{e,i}^{RTS} = p_{e,i}^{CTS} = 0$.

The successful transmission probability in a time slot of an AC is

$$\begin{aligned} P_{s,i} = P_{idle} \cdot N_i \cdot \tau_{i,idle} \cdot \prod_{z<i} \left(1-\tau_{z,idle}\right)^{round[N_z \cdot r(z,i)]} \times \\ \times \left(1-\tau_{i,idle}\right)^{N_i-1} \cdot \prod_{z>i} \left(1-\tau_{z,idle}\right) N_z + \\ + \left(1-P_{idle}\right) \cdot N_i \cdot \tau_{i,busy} \cdot \left(1-\tau_{i,busy}\right)^{N_i-1} \times \\ \times \prod_{z>i} \left(1-\tau_{z,busy}\right)^{N_z} \end{aligned} \tag{12}$$

The above are funtions of the frame error rate (FER) $p_{b,i}^{k}$, where k is either the length of the MAC and PHY header, or the length of the RTS, or CTS or ACK. Since errors in the payload are not recognised in the MAC layer but in upper layers, there is no reason to calculate the Mean packet length, as found in [15]. Similarly to the case of the payload in the MAC load, the same happens in th PHY layer. Thus an error can occur either in an error report sent from the error check of the PHY layer to the MAC layer through the PLCP header, or from an error in the MAC header recognised from the FCS (take notice that even if CRC fails to decode correctly, which is a very small possibility, the upper layer is responsible to resend the corrupted packet).

2) Mean Backoff Duration: $E[BD]_i$ is defined as the mean backoff delay, which is the summation of the backoff transitions $E[X]_i$ when the channel is idle, and the delay due to freezing $E[F]_i$, all of which referring to each AC.

$$E[BD]_i = E[X]_i \sigma + E[F]_i \tag{13}$$

The backoff transition delay is defined $E[X]_i$ as the number of slot times k that are needed, for the AC to reach state 0 and transmit, considering that the counter is at the state $b_{i,1,j,k}$ or $b_{i,0,j,k}$. The number of times the counter is stopped (freezed) are not taken into account as they are calculated seperately in equation (17).

$$E[X]_i = \frac{\sum_{j=0}^{L_i} \sum_{k=0}^{Wi,j-2} k b_{i,0,j,k}}{P_{idle}} \tag{14}$$

After some algebra the backoff transition delay is

$$E[X]_i = \frac{b_{i,0,0,0} \cdot M_i}{12 P_{idle}} \tag{15}$$

$$M_i = \sum_{j=1}^{L_i} \psi_{i,j} (W_{i,j}-1)(W_{i,j}-2)(4W_{i,j}-3) \tag{16}$$

and the delay due to freezing of the backoff counter is calculated as follows. Note that the denominator of $E[F]_i$ is the exact opposite of the denominator of $E[X]_i$.

$$E[F]_i = \frac{E[N_f]_i}{1-P_{idle}} \left[\sum_{i=0}^{3} P_{s,i} T_{s,i} + P c T_{c,i}\right] \tag{17}$$

Where $E[N_f]_i$ is the number of freezes, and is analysed as the fraction of the mean value of the counter $E[X]_i$ divided with the mean consecutive number of idle slots, defined in (10).

In order to find the MAC delay, the mean delay must be subtracted from the dropping delay defined as.

$$\begin{aligned} E[Drop]_i &= b_{i,0,0,0}(T_c + T_{protect}) \psi_{i,L_i} \times \\ & \times \left[\frac{1+p_{i,0}\left(W_{i,L_i}-1\right)}{1-p_{i,1}} p_{i,1} + \left(W_{i,j}-1\right) p_{i,0}\right] \end{aligned} \tag{18}$$

B. Elementary Conditional Probability Analysis

The proposed probabilistic analysis is simpler than the previous solution of Markov Chain, because it is based on conditional probabilities of each Access Category independently [10]. Two events are defined here. The first is called TX_i and means that a station's AC is transmitting a frame into a time slot and the second is $s = j$ is that the station's AC is in backoff stage j where $j \in [0, L_i]$, L_i is different in Basic and RTS/CTS method according to the short and long retry limit. From Bayes Theorem we have

$$P(TX_i)\frac{P(s=j|TX_i)}{P(TX_i|s=j)} = P(s_i = j) \tag{19}$$

1) Successful Transmission Probability: From equations (2)-(7) in [10], with amendements so as to include the four ACs ($i = \{0, 1, 2, 3\}$) and instead of collision probabilities, there are transmission failure probabilities, we have that the transmission probability can be written

$$\tau_i = \frac{1}{\frac{(1-p_i)}{1-p_i^{L_i+1}} \sum_{j=0}^{L_i} p_i^j \cdot (1 + E[BD]_{i,j})} \tag{20}$$

In order to include the freezing of the bacoff counters a differentiation must be made. The interruption of the backoff period of the tagged station can occur by three different events and is analyzed as follows. The first one is the collision of two or more stations, the second is the transmission of a single station other than the tagged one and the third is the transmission of a single station and the packet is corrupted. p_i is the probability that the tagged station is interrupted from the transmission of any other station (one or more)

$$p_i = 1 - \prod_{z \geq i} (1 - \tau_z)^{N_{m,z}} \tag{21}$$

and $N_{m,z} = N_z - \delta_{m,z}$ ($\delta_{m,z}$ is the Kronecker function [16]).

Whereas the probability that the tag station is interrupted by the transmission of a single station (one exactly) is given by

$$p_i' = \binom{N_i - 1}{1} \cdot \tau_i \cdot (1 - \tau_i)^{N_i - 2} \prod_{z > i} (1 - \tau_z)^{N_z} \tag{22}$$

2) Mean Backoff Duration: In this model a new approximation is incorporated for calculating the $E[BD]_{i,j}$, since in [10] the Mean Value is firstly supposed as the mean value of the Backoff Duration without freezings of Backoff Counter. Then incrorrectly the authors after the solution of their model, try to find a new Mean Value.

The phenomenon that the slot is interrupted from a collision or a successful transmission are derived from

$$\text{P(col lision|slot is interrupted)} = p_{c,i} = \frac{p_i - p_i'}{p_i} \tag{23}$$

$$\text{P(successful by one AC|slot is interrupted)} = p_{t,i} = \frac{p_i'}{p_i} \tag{24}$$

The freezing probability in each time slot is given by

$$BD_{i,j} = \frac{p_{t,i} T_{s,i} + p_{c,i} T_{c,i}}{P_{idle}\sigma + \sum_{i=0}^{3} P_{s,i} T_{s,i} + P c T_{c,i}} \tag{25}$$

$T_{c,i}$, $T_{s,i}, \sigma$ and similar values could be found in [7] and differentiate mainly in $AIFS[i]$ and probability that the current slot idle is

$$P_{idle} = (1 - \tau_i)^{N_i} \tag{26}$$

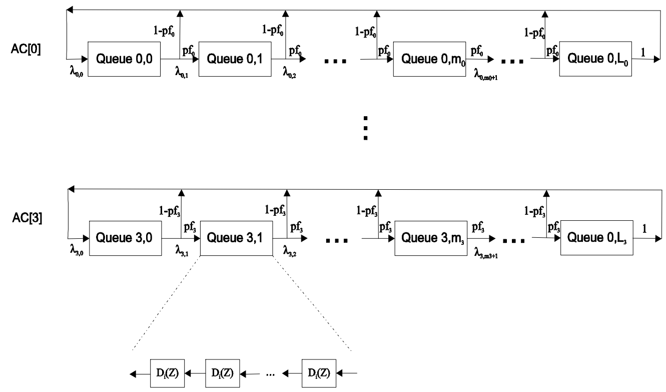

Fig. 3. Queueing network model and analyzind the Backoff Duration with Z-transform

The mean delay of the backoff duration of each backoff window is

$$E[BD]_{i,j} = \begin{cases} \sum_{k=0}^{CW_{i,j}-1} \frac{BD_{i,j}}{CW_{i,j}} & 0 \leq j \leq m_i \\ E[BD]_{i,m_i} & m_i \leq j \leq L_i \end{cases} \tag{27}$$

Where $\Delta_i = AIFS[i] - AIFS[x]$. H is the Heaviside Funtion which is used due to the difference in one slot of the backoff duration after successful transmission or after a collision (see above that the state {i,1,0,0} is missing from the Markov Chain).

C. Queueing network model analysis

This analysis is based on the Choi et al [11] queueing model. In our model the approach towards the network is different than any one proposed before, because it models the behaviour of each AC, which contains N_i stations, instead of a single station independently. Except from that, each Backoff Stage is modeled by a $G/G/\infty$ queueing system. The infinite number of parallel servers are used so that each queue can serve all stations simultaneously without queueuing delay. In addition the queueing delay is found from Z-transform. Similarly to the previous two models, the first queue has less length than the other ones. This solution is based on the assumption that the transmission probability can be expressed as the total attempt rate λ_i, divided by the number of stations of each AC independently.

$$\tau_i = \frac{\lambda_i}{N_i} \tag{28}$$

Let us define $\lambda_{i,j}$ the arrival rate and $\mu_{i,j}$ the average service rate, at each queue of each AC where $\mu_{i,k}$ is found from the Backoff Duration of each queue, which is calculated from the Z-tranform of each Queue given below. From Little's theorem the number of stations in each queue and in each AC can be found by

$$N_{i,j} = \frac{\lambda_{i,j}}{\mu_{i,j}} \tag{29}$$

The transition probability from one queue to the next one is related the arrival rates. However, it should be noted that a small difference is found from queue 0 to queue 1, as it has been explained that the value 0 of the first backoff window is not chosen.

$$\lambda_{i,j+1} = p_i \lambda_{i,j} \qquad j = 0, ..., L_i - 1 \tag{30}$$

where the total attempt rate λ_i is given by

$$\begin{aligned} \lambda_i &= \sum_{j=0}^{L_i} \lambda_{i,j} = \lambda_{i,0} \sum_{j=0}^{L_i} p_i^j \\ &= \lambda_{i,0} \frac{1 - p_i^{L_i+1}}{1 - p_i} \end{aligned} \tag{31}$$

the average service rate of each queue is found from

$$\mu_{i,j} = \frac{1}{1 + E[BD]_{i,j}} \tag{32}$$

the reason for adding 1 with $E[BD]_{i,j}$ is that to get out of the queue one more slot is need for transmission.

1) Successful Transmission Probability: Having calculated

$\lambda_{i,j}$ and $\mu_{i,j}$ we can use again Little's theorem

$$\begin{aligned} N_i &= \sum_{j=0}^{L_i} N_{i,j} = \sum_{j=0}^{L_i} \frac{\lambda_{i,j}}{\mu_{i,j}} = \\ &= \lambda_{i,0} \sum_{j=1}^{L_i} p_i^j \left(1 + E[BD]_{i,j}\right) \end{aligned} \tag{33}$$

In equation (33) the sum is very complicated to be solved and it needs computer mathematical tools. Finally τ_i is computed from equation (28).

$$\tau_i = \frac{\lambda_i}{N_i} = \frac{1}{\frac{1 - p_i}{1 - p_i^{L_i+1}} \sum_{j=0}^{L_i} p_i^j \cdot (1 + E[BD]_{i,j})} \tag{34}$$

From the above mathematical results we can see that equations (20) and (34) are the same. So both type of solutions give similar results.Thus to find the probability of successful transmission in both models we use

$$P_{s,i} = N_i \cdot \tau_i \cdot \prod_z (1 - \tau_z)^{N_{m,z}} \tag{35}$$

In order to extract the errors the same formula as in **??** must be used.

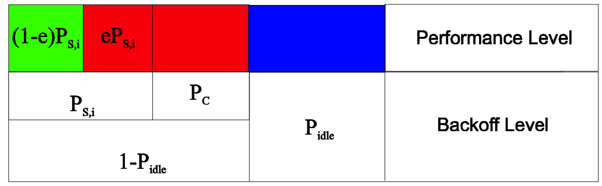

Fig. 4. Probability spaces in the analysis of 802.11 standards

2) Mean Backoff Duration: Having supposed that the standard is divided in integer Time Slots then Z-transform can be used to calculate the delay. Each state of Backoff Duration is said to have a delay $SD_i(z)$. In order to countdown to the next state, the slot must remain idle, which is sympolized by the duration of the empty in Z-transform multipled with the propability of the slot to be idle $P_{idle}Z^{\sigma}$. Hense the Z-transform of that delay is

$$SD_i(z) = \frac{P_{idle}Z^{\sigma}}{1 - p_i \cdot (p_{t,i}Z^{T_{s,i}} + p_{c,i}Z^{T_{c,i}})} \tag{36}$$

Then the total delay of Backoff Duration is given from the geometric sum, since its state is chosen uniformaly. Take notice that the first Queue of each AC, is smaller since the first state is not chosen. Using the expression 27 and raising its time to the relative time power, we have the z-transform of the Backoff Duration of each prior to a collision. The mean value is taken after differentiation of $BD_{i,j}(z)$ and the dumny parameter set $z = 1$.

$$E[BD]_{i,j} = BD_{i,j}^{(1)}(1) = \frac{W_{i,j} - H[j-1]}{2} A \tag{37}$$

where A is the additional quantity due to freezings of the backoff counter.

$$A = \frac{(1 - p_i') \cdot \sigma + p_i' T_{s,i} + (p_i - p_i') T_{c,i}}{(1 - p_i')}$$

III. Throughput and Delay

A. Saturation Throughput

1) Block-ACK Disabled: The saturation throughput for

every AC and for packets with mean length $E[L]$ is given by the

$$S_i = \frac{P_{s,i} E[L]}{T_{slot,i}} \tag{38}$$

where

$$T_{slot,i} = P_{idle}\sigma + \sum_{i=0}^{3} P_{s,i}T_{s,i} + PcT_{c,i} + A_{Error,i} \tag{39}$$

In the following equations $P_{E,i}^{x}$ are the probability of transmission errors taking into account the frame has not collided.

$$A_{Error,i} = P_{E,i}^{data} T_{E,i}^{data} + P_{E,i}^{ACK} T_{E,i}^{ACK} + P_{E,i}^{RTS} T_{E,i}^{RTS} + +P_{E,i}^{CTS} T_{E,i}^{CTS} \quad (40)$$

the correct order for calculating the $P_{e,i}$ since the RTS is transmitted first then the CTS, the data follows and lastly the ACK. Seting the values $k = RTS$, $y = CTS$, $z = data$ and $v = ACK$

$$P_{E,i}^{k} = P_{s,i} \cdot p_{e,i}^{k} \quad (41)$$

In order to have an error in the CTS, no errors should have occured in the RTS

$$P_{E,i}^{y} = P_{s,i}(1 - p_{e,i}^{k}) \cdot p_{e,i}^{y} \quad (42)$$

Similarly to the previous case, in order to exchange data frames, and thus a data packet to be corrupted the RTS/CTS should have exchanged without errrors.

$$P_{E,i}^{z} = P_{s,i} \cdot (1 - p_{e,i}^{k}) \cdot (1 - p_{e,i}^{y}) \cdot p_{e,i}^{z} \quad (43)$$

$$P_{E,i}^{ACK} = P_{s,i} \cdot (1 - p_{e,i}^{x}) \cdot (1 - p_{e,i}^{y}) \cdot (1 - p_{e,i}^{z}) \cdot p_{e,i}^{v} \quad (44)$$

The collision probability thus is

$$Pc = 1 - P_{idle} - \sum_{i=0}^{3} \left(P_{s,i} - P_{E,i}^{data} \right) \quad (45)$$

We must also mention that whenever the retry limit is ended the packet is dropped. However such a probability is inlcuded in Pc and the retransmissions required after a collision or a drop are based on the upper layer and does not affect the performance of the studied MAC layer.

2) Block-ACK Enabled: Another characteristic of the IEEE 802.11e standard is Block-ACK feature, which not is obligatory. However Block-ACK can mitigate the overhead problem especially in higher data rates which are supported by the forthcoming 802.11n. Data Rates of nearly 432Mbps tend to have 10% of MAC efficiency [20]. The Block-ACK feature allows a number of data units to be transmitted and afterwards the sender sends a Block ACK request (BAR) and receives a Block ACK (BA) frame. Throughput is increased since less ACK frames are used for a transmission. Analysis of the Block ACK scheme (BTA) is not in the scope of the paper and more information can be found in the standard [2]. The problem with errors in the BTA scheme is similar to the RTS/CTS and requires to change all the above equations which include errors in RTS and CTS frames with errors in BAR and BA frames and to make all the respective errors of ACK equal to zero. However since the exchange of frames in a transmission period of RTS/CTS is prior of the data frames, whereas the BAR and BTA frames are exchanged afterwards, the equations that should be changed are (41-43) with $k = data$, $y = BAR$ and $z = BA$.

Where $T_{E,i}^{r} = r + H + \delta$, $r = BAR$ or BA. Take notice that (40) here is $A'_{Error,i}$, because $P_{E,i}^{data}$ is not the same as in (43). Finally

$$S'_i = \frac{P'_{s,i} \cdot F \cdot E[L] + P'^{data}_{E,i} \cdot ER[L]}{P_{idle}\sigma + \sum_{i=0}^{3} P'_{s,i} T_{s,i} + PcT_{c,i} + A'_{Error,i}} \quad (46)$$

Supposing Gaussian like erroneous channel we have that the $ER[L]$, which is part of $E[L]$, and is defined as the transmitted frame size in the BTA scheme and is given by the binomial PMF of existing e errors in a group of F MPDUs (Requested Block Size). But since we are interested in mean values we have that

$$ER[L] = F \cdot p_{e,i}^{k} \cdot E[L] \quad (47)$$

with $k = F * E[L]$ in (**??**).

The time for successful transmission $T_{s,i}$ thus is much bigger since it includes F frames and SIFS time, plus the exhange of the BAR and BA. Moreover H is the Physical Layer Header and δ the transmission delay.

$$\begin{aligned} T_{s,i}^{basic} &= T_{E,i} = F \cdot (H + E[L] + SIFS + \delta) + \\ &+AIFS[i] + H + T_{BAR} + SIFS + \delta \\ &+H + T_{BA} + \delta \end{aligned} \quad (48)$$

$$\begin{aligned} T_{c,i} &= F \cdot (H + E[L] + SIFS + \delta) + \\ &+EIFS[i] + H + T_{BAR} + \delta \end{aligned} \quad (49)$$

where $EIFS[i] = SIFS + H + T_{BA} + AIFS[i]$.

B. Mean Delay

1) Mean Value of the MAC delay: In 802.11e [2] two different access mechanism are provided. The first one is with the use of acknowledgements ACK (or else basic) and the other by transmitting Request To Send and Clear To Send packets. The transmission times $T_{s,i}^{Basic}$ and $T_{s,i}^{RTS/CTS}$, and the times $T_{c,i}^{Basic}$ and $T_{c,i}^{RTS/CTS}$ for a collision can be found in [7].

Mean delay can be defined for each AC by the following equation.

$$\begin{aligned} E[D]_i &= E[N_{cs}]_i (E[BD]_i + T_c + T_{protect}) + \\ &+E[BD]_i + T_{s,i} \end{aligned} \quad (50)$$

The first part of the equation is the delay due to consecutive unsuccessful transmissions, the second part is the mean backoff delay, whenever this transmission shall be completed and the third part is the transmission duration. All are referred to each AC. Following the above equation, $E[N_{cs}]_i$ can be defined as the mean number of collisions that are followed by a successful transmission.

$$E[N_{cs}]_i = \frac{1 - P_{idle} - P_{s,i}}{P_{s,i}} \quad (51)$$

2) MAC delay PGF Modeling: From the equation (37), the mean value, the varriance and the MAC delay distribution can be found. However in the previous subsection we have shown a unified method to find the mean MAC delay for all the models, in this subsection the above metrics for MAC delay will correspond only to the model #3. This happens because the solution of the Markov Chains after theoritically infinite retries gives mean values. Thus the Z-transform of the MAC delay will be given as a funtion of $D_i(z)$ (**??**).

$$\begin{aligned} D_i(z) \quad &= \quad (1-p_i)\, z^{T_{s,i}} \sum_{j=0}^{L_i} \left[\left(p_i z^{T_c}\right)^j \prod_{f=0}^{j} BD_{i,f}(z) \right] + \\ &+ \left(p_i z^{T_c}\right)^{L_i+1} \prod_{f=0}^{L_i} BD_{i,f}(z) \end{aligned} \tag{52}$$

The first part signifies the correct transmission $\left[(1-p_i)\, z^{T_{s,i}}\right]$ having encountered a number of collisions in the previous stages, whereas the second part is the delay associated with dropping of a packet after $L_i + 1$ retries. However to find the mean value and the varriance, the 1^{st} and the 2^{nd} moments of the above equations must be found, respectively.

$$E[D]_i = \frac{\partial D_i(z)}{\partial z}|_{z=1} \tag{53}$$

$$Var[D]_i = \frac{\partial^2 D_i(z)}{\partial z^2}|_{z=1} + \frac{\partial D_i(z)}{\partial z}|_{z=1} - \left\{ \frac{\partial D_i(z)}{\partial z}|_{z=1} \right\}^2$$

The last part is to find MAC delay distribution. It is well-known that every Z-transform can be written as

$$D_i(z) = \sum_{k=0}^{\infty} d_{i,k} z^k$$

It seems that from the definition, $d_{i,k}$ is the inverse Z-transform of the $D_{i,j}(z)$. A method that gives the inverse Z-transform with a predefined error bound is the Lattice-Poisson Algorithm [17], with is valid $|d_{i,k}| \leq 1$.However in the situation of $D_{i,j}(z)$, $d_{i,k}$ is a PDF and thus validates the above method. Thus the PDF of the MAC delay is given by

$$d_{i,k} = \frac{1}{2kr^k} \sum_{h=1}^{2k} (-1)^h \operatorname{Re}\left(D_{i,j}\left(re^{\frac{i\pi j}{k}} \right) \right) \tag{54}$$

In order to validate the above PDF, a variety of OPNET simulations has been used. Thus a heuristic approach has been found which is closely related with (54).

3) Queueing Delay: We consider a simple queueing system, namely the $M/G/1/$ with infinite size, where the 1 corresponds to the wireless channel (we have supposed that a simple queue can serve all the ACs). For the mean queueing delay, $E[MQ]_i$, we have from the *Pollaczek-Khinchine's formula [18].*

$$E[MQ]_i = \frac{\rho_i \cdot E[D]_i}{2\,(1-\rho_i)} \varepsilon \tag{55}$$

where $\rho_i = \lambda_i \cdot E[D]_i$ and

$$\varepsilon = 1 + \frac{E[D]_i^2}{Var[D]_i} \tag{56}$$

From the Little's theorem the expected number of packets in the queue can be found

$$Q_i = \lambda_i \cdot E[MQ]_i \tag{57}$$

However the above cases are for Poisson arrivals. A better approximation is to suppose that either the arrival process or the service time is Markovian and model the network with a $GI/G/1$ queue. With this type of queue the *Marchal's approximation* [19] gives the mean value of the queueing delay, $E[MQ]_i'$ from 55 by substituing ε with

$$\varepsilon' \approx \left\{ \frac{\upsilon_{\rho,i} + Var[D]_i}{E[D]_i^2} \right\} \left\{ \frac{E[D]_i^2 + Var[D]_i}{\alpha^2 + Var[D]_i} \right\} \tag{58}$$

4) Total Delay: The total delay is the sum of the Queueing delay and the MAC delay. The sum is possible since is the mean value is a linear operator.

$$E[T]_i = E[D]_i + E[MQ]_i \tag{59}$$

IV. Comparison Analysis and Results

For validating the correctness of the mathetical analyses, OPNET modeler^{TM} (version 12) was used with the EDCA simulation model incorporated. The channel capacity was set to 1Mbps, packet length was 1024 bytes and the interarrival time was set according to saturation conditions. Our simulation was performed in a rectangular grid of 100mX100m, with single-hop transmissions.

In Fig.4 saturation throughput comparison is presented for basic access method and transmission mode (1Mbps) and without having Frame Error Rate (BER). However starvation of low AC2 and AC3 happen and they get very low values when in saturation. Since this phenomenon, in the rest of the figures both of them will be omitted because insignifigant varriations of throughput is not part of the paper. Similar study but for RTS/CTS access method is given in Fig.5.

In Fig.6 and Fig.7 comparison of different values of FER is done for Basic and RTS/CTS access method. It should be also mentioned that CRC identifies errors in packets offered in higher layers. However it is worth seeing that in RTS/CTS method the degradation of throughput stays in very low values, and shows that RTS/CTS transmission can be a solution in erroneous environments. The comparison of the three models in these graphs is avoided since the degradation of the performance due to Gaussian errors seems to linear to FER changes.

In both figures 6 and 7, the small difference in delay is because in simulations, the queue must be set in such a value

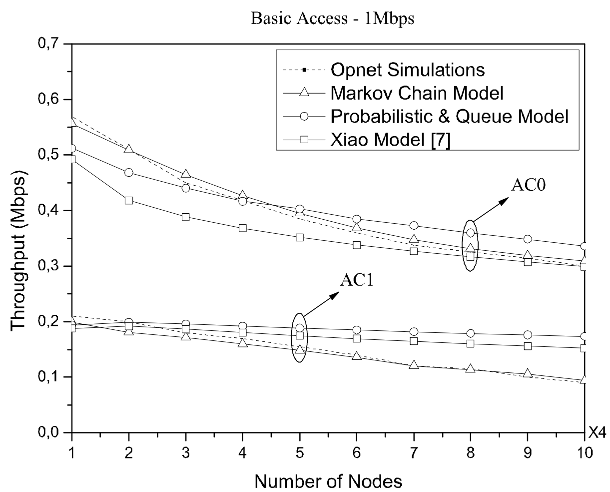

Fig. 5. Model compariston for saturation throughput using basic access method in 1Mbps bandwidth

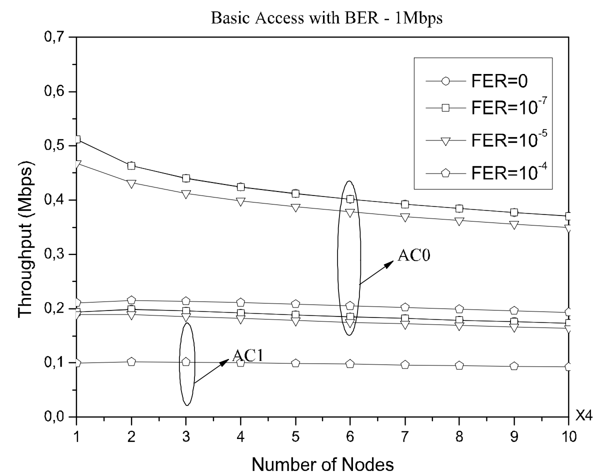

Fig. 7. Saturation Throughput using Basic Access Method in 1Mbps bandwidth, and varriable varriable Bit Error Rates

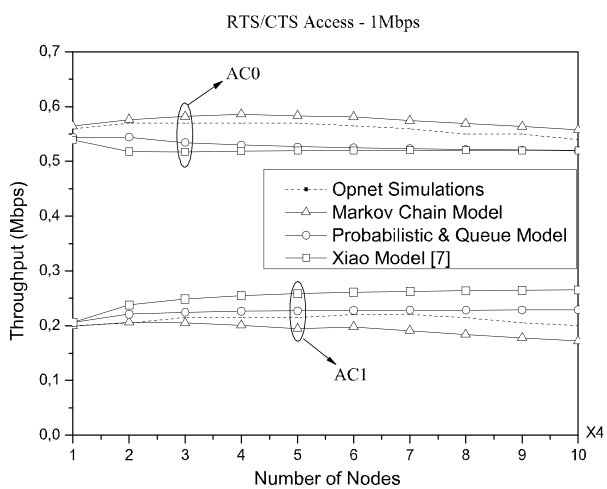

Fig. 6. Model comparison for saturation throughput using RTS/CTS access method in 1Mbps bandwidth

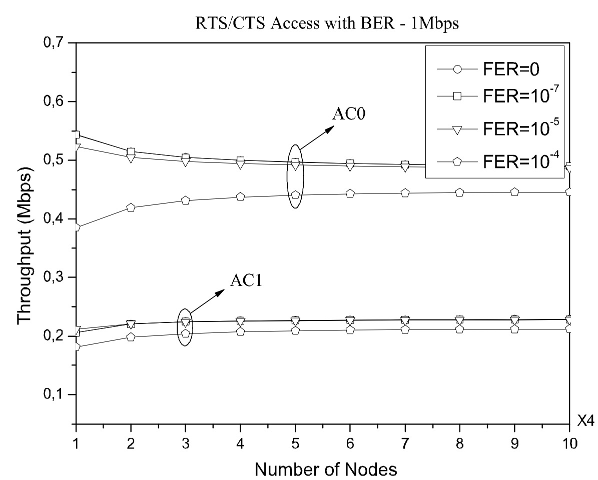

Fig. 8. Saturation Throughput using RTS/CTS Access Method in 1Mbps bandwidth, and varriable varriable Bit Error Rates

that corresponds to the exact saturation value. If the generation of packets in the simulator is higher than the transmission rate, then either the packets shall be dropped or the delay values will increase due to queueing delay. However to set such a saturation value is relatively hard. Similar problem does not affect the saturation throughput.

In Fig.6 IEEE 802.11a/e is modeled with a bandwidth of 24Mbps (analytical and simulation set) and as it is seen when enabled the Block-Ack mechanism can offer higher throughput in higher load and can even provide better results in higher bandwidth occasions. This is due to the reduction of unecessary ACKs. The reason for modeling IEEE 802.11a is that higher bandwidth are going to be used in 802.11n which physical and MAC layer that do not change significantly. The above equations include the changes due to different transmission rates of 802.11a.

The performance comparison of the three models is shown in various conditions. We can see that the model based on the Markov Chain analysis with the incorporation of the previous slot are more accurate to simulation results. However something not shown in the figures is the complexity of each solution. Although model #1 seems to have better results compared to the other models, the mathematical complexity

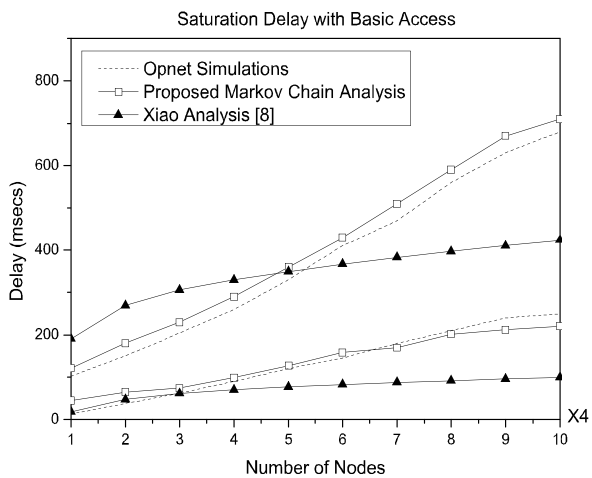

Fig. 9. Analysis of MAC Delay of the two higher ACs under saturation condition and basic access mode.

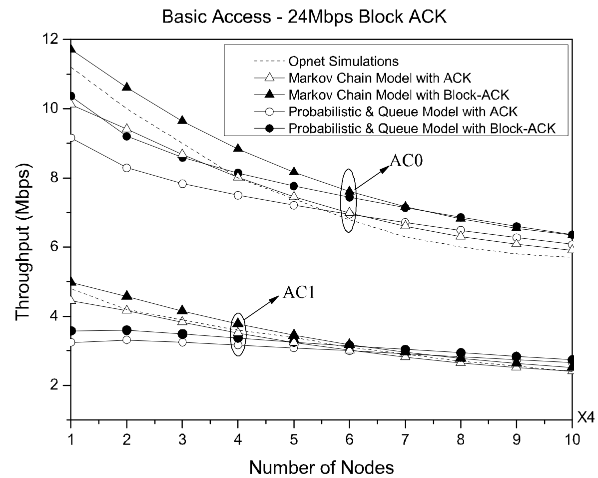

Fig. 11. Model comparison of saturation throughput with Block ACK enabled (F=64) in 24Mbps with IEEE 802.11a

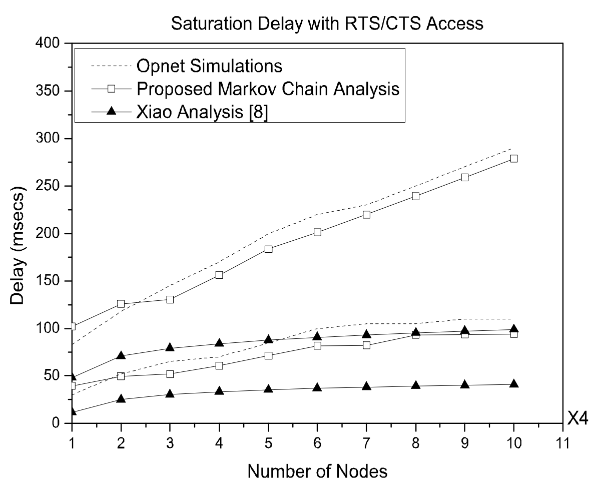

Fig. 10. Analysis of MAC Delay of the two higher ACs under saturation condition and RTS/CTS access mode.

of the solution is higher due to the independence of each state, which models a state of the BEB, and to the correlation with the state of the previous slot. However the state of the previous slot is hardly incorporated in models based on queueing theory or geometric distribution since it does not allow the flexibility to change Backoff Duration according to the simulation needs. A significant drawback of the proposed Markov Chain is that non-saturation throughput analysis becomes a complex problem, whereas in the other analyses the arrival rate could be changed very easily with simple algebra. On the other hand, the modeling of independent states makes easier to provide amendments in the analysis, such as the one given with the inexistance of the first state. Take notice that if the delay analysis of the probabilistic model was used in the queueing theoritic modeling then the results would much. Since that we can validate that these analyses can provide correct results if the hypotheses of the model are correct and after the use of a simulator becomes a need to validate the hypotheses and the totality of the analysis. The Markov Chain Model of Xiao [7] was also analyzed and proved to provide poorer results compared to the new models which take into account the proposed features. In fact except from the three already analyzed solutions we can also Lastly it seems that after a Gaussian $FER = 10^{-4}$ the performance of the network is degraded significantly and in a great number of nodes the performance stays the same. We can easily sum up that the general contibution of this work is that these analyses, are the material for the analysis of any new standard.

A. Complexity Analysis

Complexity is an important characteristic as regards mathematical analysis and algorithms. Comparing the three approaches in terms of complexity allows an insight in the usability and scalability of each one. The Markov model is obviously the most complex one. It is easy to observe that the analysis of this first model requires big Markov chains and more mathematical formulas to be calculated. Moreover, the addition of extra features and the incorporation of realistic modelling in this approach injects even more complexity in the final calculations. Appart from this heuristic approach, a computational complexity comparison can be performed in terms of big-o notation. Instead of computer instructions we use a simple formula calculation as the basic unit of complexity. Each algorithm's order of complexity can be estimated as a function of the number of calculation points N, the number of steps used in the fixed point iteration method M, the retry limits L_i and the number of ACs calculated i. In Table

I of the appendix, the results show that all three algorithms have linear complexity relative to M and N, and that the Markov model is approximately four times more complex than the other two approaches.

B. Accuracy Analysis

V. Discussion and Conlusion

The purpose of this work is threefold. Present a comparison analysis of the most known analyses in order to find the best method to numerically analyze the standard, while setting the pace for future methods of analysis. In addition the Modeling techniques, based on complexity and accuracy theory, have not been studied before in wireless networks and they could be a field of great interest, since the computer resources are finite. The second goal was to extend the already known analyses, introducing features that change considerably the performance analysis. The combination of such features optimizes the MAC protocol outcomes, and makes each mathematical analysis avant-garde by itself. The third goal was to correct the IEEE 802.11e from general misunderstandings, such as the phenomenon of not providing instant access after a successful transmission and the dependence of the queuing delay from the MAC delay, and thus correcting the formula of the total delay.

Each one of the above goals carries by themselves scientific interest and is combined with Gaussian erroneous channel. Other fading channels can be used in order to optimize MAC layer metrics through cross layer techniques. In addition Z-transform is used as a method to propose the accurate delay distribution, while having different types of queues, depicting the heterogeneity of the multimedia applications. Even if such combination of features and corrections increase the complexity of the proposed analyses, it is the price that must be paid for efficacy and outcome improvements, epsecially when new machines can solve problems with great approximation. Opnet Modeler 12, was used in two ways. Firstly to verify the correctness of analyses and as a tool to calulate the accyracy analysis.

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TABLE I

Big-O notation for computational complexity of the compared models.

	τ	$E[BD]$	both τ and $E[BD]$
Markov chain	$Com1$	$Com2$	$Com2$
Elementary conditional probability	$Com3$	$Com3$	$Com3$
Queueing	$Com3$	$Com3$	$Com3$

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VI. Appendix

For the complexity analysis we have the above formulas

$$Com1 = O\left[NM\left(\sum_i (4L_i+1)\right)\right]$$

$$Com2 = O\left[N\left(M\left(\sum_i (4L_i+1)\right)+\sum_i L_i\right)\right]$$

$$Com3 = O\left[NM\left(\sum_i (L_i+3)\right)\right]$$

Ordinary values for the parameters are: $N=10$, $M=20$, $L=7$ and $i=4$.