Cell Capacity for IEEE 802.16 Coverage Extension

(Invited Paper)

Georgios S. Paschos*, Petteri Mannersalo* and Thomas Michael Bohnert†

†VTT Technical Research Centre of Finland, P.O. Box 1000, FI-02044 Finland
Email: gpasxos@ieee.org, petteri.mannersalo@vtt.fi
†Center of Informatics and Systems, University of Coimbra
Email: tmb@nginet.de

***Abstract*—This paper analyzes a two-hop extension to the coverage of an IEEE 802.16 cell. There is natural degradation in cell capacity due to multihop communications which can be mitigated by spatial reuse, adaptive modulation and coding. We estimate the available capacity by analyzing the random geometry related to the locations of the base station, the sponsor nodes and the mesh subscriber stations situated two hops away from the base station. The results show the trade-offs of extending the coverage area and the decrease of the capacity.**

***Index Terms*—Spatial reuse, link capacity, broadband wireless access.**

I. Introduction

New broadband wireless access technologies are promising the delivery of demanding applications like video streaming and fast data access even to mobile users. For distances beyond the range of local area networks, the typical technologies are either Internet oriented solutions or 3G/4G cellular phone systems. One of the major technologies of the former type is the Worldwide Interoperability for Microwave Access (WiMAX) which is based on the IEEE 802.16 standards [1].

Mesh networking is a part of IEEE 802.16 specification. There are numerous applications of mesh networks (see, e.g., [2], [3]). As an example, infrastructure relay nodes (sponsor nodes (SN) in 802.16 terminology) are proposed for the coverage extension [4]. This extension is useful in case of shadowing, or even for cost efficiency in the perimeter of the metropolitan network. The standard also allows non-infrastructure nodes to act as sponsor nodes.

The IEEE 802.16 standard leaves undefined the central scheduler located in the base station (BS). A number of different scheduling schemes have been proposed to promote quality of service, fairness, maximum throughput, etc. In this work, we study a scheduling strategy which exploits spatial reuse for the mesh nodes. This strategy can be applied on top of another scheduling scheme. The extra capacity due to spatial reuse partially mitigates the capacity loss resulting from multihop communications. In this paper, we study the extension of the network perimeter with 2-hop communications where the sponsoring nodes are not part of the infrastructure.

We model adaptive modulation and coding (AMC), a basic property of WiMAX networks, by an ideal Shannon channel with a gap, as introduced in [5]. The capacity of each link is then related to its distance, analogously to real WiMAX networks. Assuming a fair scheduling policy together with spatial reuse, we derive the capacity of the system under certain simplifying assumption on the geometry of the system.

The results of the paper can be used by a network operator to estimate the impact of coverage extension in a WiMAX cellular deployment: Should a new BS be installed for the extra customers, or can mesh networking provide a smooth and cost effective solution? However, using non-infrastructure nodes, i.e., customers, as relay causes many problems that most operators want to avoid. In a continuation of this work, we are studying the case where the locations of the sponsoring nodes can be freely chosen. In practice, this means that inexpensive and easy to install and transfer infrastructure relays are selectively put at the boundaries of a cellular network. When, out of perimeter, nearby customers grow in numbers, a new BS can be installed and the SNs can be moved to the new boundary area. This idea provides smooth infrastructure expansion for a cellular network.

The paper is organized as follows. The network model is described in Section II and the link and network capacity analysis is presented in Section III. Section IV focuses on the spatial reuse issues. The numerical results based on the analysis and Monte-Carlo simulations are shown and compared in Section V. The paper is concluded in section VI with some insights to future work.

II. Network Model

We study a random cellular topology of subscriber stations (SS) around a BS. The cell radius R, i.e., the transmission radius of the nodes, is considered fixed and the network nodes are in a quasi-stationary state since mobility is not of interest. The interference is modeled by a disk of radius $(1+\Delta)R$. There are $N-M$ uniformly distributed SSs inside the cell. The rest M nodes are called mesh stations (MS) and they are put outside the cell in 2-hop distances $d_i \in [R, 2R]$ away from the BS.

The principles of the cell coverage extension are visualized in Fig. 1. Each MS has an optimal relay candidate inside the cell resulting a 2-hop path with the maximal bottleneck capacity. Denote $r(i,j)$ the distance from node i to node j and number the nodes such that node 0 is the BS, nodes $i \in I_c \doteq \{1,\dots,N-M\}$ are inside the cell and the nodes $i \in I_m \doteq \{N-M+1,\dots,N\}$ are outside the cell. Then the

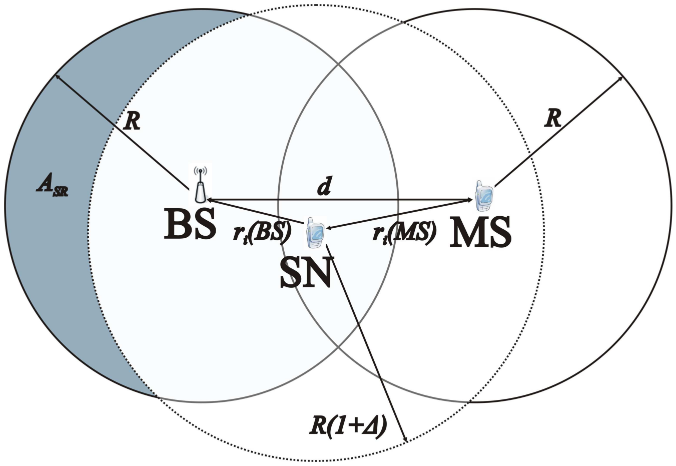

Fig. 1. Extending the network coverage with sponsor nodes.

optimal SN for MS j, with $j \in I_m$, is given by

$$\mathrm{SN}(j) = \underset{i \in I_c}{\arg\min} \max\{r(i,0), r(i,j)\}.$$

The length of the longer link, i.e., the bottleneck on the 2-hop path from node j to the BS, is given by

$$R_j^b = \max\{r(\mathrm{SN}(j),0), r(\mathrm{SN}(j),j)\}.$$

Necessarily, $R_j^b \geq d_j/2$. If $R_j^b > R$ then MS j cannot communicate with the BS by the assumption of the fixed transmission range R. This means that at least one SS must lay inside the intersection the two disks with centers at the BS and MS j in order to have a 2-hop path from the BS to MS j.

If the interference parameter Δ is small enough, then the system can take advantage of spatial reuse: The nodes located in the non-interfering set $A_{\mathrm{SR}}(\mathrm{SN}(j))$ may utilize the same OFDM slots (time&frequencies) as transmissions from MS j to $\mathrm{SN}(j)$.

III. Capacity

Adaptive modulation and coding (AMC) is modeled using the Shannon channel capacity with a SNR gap Γ. The link capacity C is then related to the distance r between the communicating nodes, i.e.,

$$C(r) = \begin{cases} BW \cdot \log_2\left(1 + \frac{\mathrm{SNR}_0}{\Gamma \cdot r_0^\alpha}\right), & \text{if } r \leq r_0, \\ BW \cdot \log_2\left(1 + \frac{\mathrm{SNR}_0}{\Gamma \cdot r^\alpha}\right), & \text{if } r_0 < r \leq R, \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

where BW is the channel bandwidth, SNR_0 is the Signal-to-Noise ratio in a reference distance, α is the path loss exponent and Γ is the SNR gap. In order to model m-ary adaptive QAM with variable coding, typical value of Γ is 8 dB. Note also that we have bounded the capacity from above to avoid problems related to (unrealistic) infinite capacities. By [6], this upper bound could be approximated to be around 7 bps/Hz.

In the following subsections, the capacities of 1-hop and 2-hop paths are found. Moreover, we derive an estimate for the overall network capacity. Although, the resulting formulae do not have closed-form expression, they all can be numerically evaluated, for example, by Mathematica.

A. 1-hop capacity

Recall that $2r/R^2$ is the probability density function for radius $|X|$ of a uniformly random point X inside disk of radius R and center at the origin. Then the expected capacity of the link between BS and SS i located 1-hop away is given by

$$\mathrm{E}\left[C_L^1(i,0)\right] = \frac{2}{R^2}\int_{r_0}^{R} C(r)\, r\, \mathrm{d}r + \mathrm{P}(r \leq d_0)\, C(d_0),$$

where $i \in I_c$, $\mathrm{P}(r \leq d_0) = \frac{r_0^2}{R^2}$ and $C(r)$ is defined in Equation (1).

B. 2-hop capacity

Now consider node $j \in I_m$ belonging to the set of the mesh nodes. The bottleneck capacity is governed by the random variable R_j^b. The tail distribution $\mathrm{P}\left(R_j^b > r\right)$ is found by the fact that if there are no nodes inside the intersection of the disks of radius r and centers at the BS and MS j, then necessarily either link from $\mathrm{SN}(j)$ is longer than r (see Figure 1 for case $r = R$). Thus

$$\mathrm{P}\left(R_j^b > r\right) = \begin{cases} \left[1 - \frac{\Upsilon(d_j,r,r)}{\pi R^2}\right]^{N-M}, & r \leq R, \\ \left[1 - \frac{\Upsilon(d_j,R,r)}{\pi R^2}\right]^{N-M}, & R < r \leq R + d_j, \\ 0, & r > R + d_j, \end{cases}$$

where

$$\Upsilon(d, r_1, r_2) = r_1^2 \arccos\left(\frac{d^2 + r_1^2 - r_2^2}{2dr_1}\right) + r_2^2 \arccos\left(\frac{d^2 - r_1^2 + r_2^2}{2dr_2}\right) - \frac{1}{2}\sqrt{\left((r_1+r_2)^2 - d^2\right)\left(d^2 - (r_1 - r_2)^2\right)}$$

is the area of the intersection between two circles of radius r_1 and r_2 at with center at distance d from each other.

We define the transmission rate for a 2-hop path by the capacity of the bottleneck link. This means that

$$C_L^2(j,0) = C(R_j^b).$$

and the expected 2-hop link capacity is given by

$$\mathrm{E}\left[C_L^2(j)\right] = \int_0^{C(d_j/2)} \mathrm{P}\left(C_L^2(j) > x\right) \mathrm{d}x$$

$$= \mathrm{P}\left(R_j^b \leq R\right) C(R) + \int_{C(R)}^{C(d_j/2)} \mathrm{P}\left(R_j^b \leq C^{-1}(x)\right) \mathrm{d}x,$$

where

$$C^{-1}(x) = \left(\frac{\mathrm{SNR}_0}{\Gamma \cdot (2^{x/\mathrm{BW}} - 1)}\right)^{1/\alpha}.$$

We can also calculate the expected two-hop link capacity conditioned to the event that the SN is actually in the range of the MS. Since

$$\begin{aligned}\mathrm{E}\big[C_L^2(j)\big] =& \mathrm{P}\big(R_j^b \le R\big)\,\mathrm{E}\big[\,C_L^2(j)\,\big|\,R_j^b \le R\big] \\ &+ \mathrm{P}\big(R_j^b > R\big)\,\mathrm{E}\big[\,C_L^2(j)\,\big|\,R_j^b > R\big]\,,\end{aligned}$$

where the latter term equals zero, we have

$$\mathrm{E}\big[\,C_L^2(j)\,\big|\,R_j^b \le R\big] = \frac{\mathrm{E}\big[C_L^2(j)\big]}{\mathrm{P}\big(R_j^b \le R\big)}.$$

C. Network capacity for fair scheduling

From the network perspective, a MS generates two bidirectional links, one for each hop. Assuming a fair scheduling policy, the MS will be assigned enough resources in order to maintain the same average throughput as the other nodes experiencing. This results in depletion of overall capacity. In practice, the BS can decide whether it is meaningful to assign the same throughput all stations (throughput fairness) or the same amount of resources (resource fairness). In this analysis we assume the former.

We assume that the network is fully saturated, i.e., each node has always something to receive and send. Under the chosen fair scheduling policy, time/frequency space is divided among the users such that each node can send D and receive γD bits during each slot. Here $\gamma > 0$ represents the asymmetry ratio between uplink and downlink. Additionally, each SN is given extra resources for relaying. The overall capacity of the network C_{nw} can be measured as the ratio of the amount data sent over one slot of the time/frequency space (excluding the relay traffic) and resources needed for that (including the relay traffic). Using the symmetries of the links, i.e., $C(r(i,j)) = C(r(j,i))$ for all $i \neq j$, we get

$$\begin{aligned}C_{\mathrm{nw}} &= \frac{N(1+\gamma)D}{\sum\limits_{i\in I_c} \frac{(1+\gamma)D}{C(r(i,0))} + \sum\limits_{j\in I_m}\left(\frac{(1+\gamma)D}{C(r(\mathrm{SN}(j),0))} + \frac{(1-\beta_j+\gamma)D}{C(r(\mathrm{SN}(j),j))}\right)} \\ &= \frac{N}{\sum\limits_{i\in I_c} \frac{1}{C(r(i,0))} + \sum\limits_{j\in I_m}\left(\frac{1}{C(r(\mathrm{SN}(j),0))} + \frac{(1-\frac{\beta_j}{1+\gamma})}{C(r(\mathrm{SN}(j),j))}\right)}\end{aligned}$$

where $\beta_j \in [0,1]$ denotes the percentage of effective spatial reuse for the uplink from node j to SN(j). In our model, β_j is the proportion of the uplink capacity which can scheduled simultaneously to MS j and nodes located in the non-interfering set $A_{\mathrm{SR}}(\mathrm{SN}(j))$ (see Figure1).

If some of the mesh nodes do not have a sponsor node, the previous formula is useless because our definition gives zero capacity. Thus we need to condition with respect that all mesh stations have at least one potential sponsor node, i.e., $R_j^b \le R$. Note that the probability of this event, i.e., $\mathrm{P}\big(R_j^b \le R\big)$, gets very small when $d_j \approx 2R$. In such case, the conditioning will bias the results strongly.

To make analysis simpler and more transparent, from now on we assume that

1) all mesh nodes are located exactly at the same point, d away from the base station,
2) and that there is at least one node which can act as a common relay to all these mesh nodes.

Moreover, denote $d \doteq d_j$, $R^b \doteq R_j^b$, $\beta \doteq \beta_j$ and $\mathrm{SN} = \mathrm{SN}(j)$ for $j \in I_m$.

Now the expected network capacity can be approximated as follows:

$$\begin{aligned}&\mathrm{E}\big[\,C_{\mathrm{nw}}\,|\,R^b \le R\big] \\ &\approx \frac{N}{\mathrm{E}\Big[\frac{N-M}{C(r(i,0))}\,\Big|\,R^b \le R\Big] + \mathrm{E}\Big[\frac{M\left(2-\frac{\beta}{1+\gamma}\right)}{C(r(\mathrm{SN},0))}\,\Big|\,R^b \le R\Big]} \\ &\approx \frac{N}{\mathrm{E}\Big[\frac{N-M}{C(r(i,0))}\Big] + \frac{(2+2\gamma-\mathrm{E}[\beta\,|\,R^b\le R])\mathrm{E}\Big[\frac{M}{C(r(\mathrm{SN},0))}\,\Big|\,R^b\le R\Big]}{1+\gamma}},\end{aligned} \tag{2}$$

where we have used that $r(\mathrm{SN}(j),j)$ and $r(\mathrm{SN}(j),0)$ are equally distributed by the symmetry. Since the detailed analysis of the dependence between β and $r(\mathrm{SN},0)$ is beyond the scope of this paper, we assume them independent (which, of course, is not true). We also neglect the condition $\{R^b \le R\}$ when calculating the expectation of $1/C(r(i,0))$. Whenever $N - M$ is large this approximation is well motivated.

Analogous to 1-hop link capacity, we have

$$\mathrm{E}\bigg[\frac{1}{C(r(i,0))}\bigg] = \frac{2}{R^2}\int_{r_0}^{R} \frac{1}{C\,(r)}\,r\,\mathrm{d}r + \mathrm{P}(r \le d_0)\,\frac{1}{C\,(r_0)},$$

where $\mathrm{P}(r \le r_0) = \frac{r_0^2}{R^2}$. Moreover, one can show that

$$\begin{aligned}&\mathrm{E}\bigg[\frac{1}{C(r(\mathrm{SN}(j),0))}\,\bigg|\,R^b \le R\bigg] \\ &= \int_{\frac{-2R+d}{2}}^{\frac{2R-d}{2}} \int_0^{\sqrt{R^2-\left(\frac{d}{2}+|x|\right)^2}} \frac{f(x,y,d,M-N)}{C\left(\sqrt{(x+\frac{d}{2})^2+y^2}\right)}\,\mathrm{d}x\,\mathrm{d}y,\end{aligned}$$

where

$$\begin{aligned}&f(x,y,d,n) \\ &= \frac{2}{\Upsilon(d,R)}\left(1-\frac{\Upsilon\left(d,\sqrt{(\frac{d}{2}+|x|)^2+y^2}\right)}{\pi R^2}\right)^{n-2} \times \\ &\left(1-\frac{n\Upsilon\left(d,\sqrt{(\frac{d}{2}+|x|)^2+y^2}\right)}{\pi R^2}+\frac{(n-1)\Upsilon(d,R)}{\pi R^2}\right)\end{aligned}$$

and $\Upsilon(d,r) \doteq \Upsilon(d,r,r)$. In this paper, $\mathrm{E}\big[\,\beta\,|\,R^b \le R\big]$ is estimated only by simulation.

IV. Spatial Reuse

The main idea of spatial reuse is to allow simultaneous transmissions from nodes that do not share common receivers. Scheduling with spatial reuse for wireless networks was originally introduced in [7] and restudied in terms of transport capacity and connected to wireless network topology in [8]. Scheduling with spatial reuse in 802.16 mesh networks has been considered, for example, in [9], [10].

In the current paper, we are focusing on the capacity of a cell with extended coverage and a throughput fair scheduling strategy. Moreover, we propose a cognitive way to schedule transmissions utilizing spatial reuse. Under high load, i.e., when the capacity of the cell is actually affected by the 2-hop communications, the scheduler tries to reassign OFDMA symbols used by 1-hop SS-BS uplink communications to the link from the MS to the SN, while avoiding the interference between the links. Spatial reuse in the downlink is not studied because the current standards regarding centrally coordinated mesh topologies are built in such way that only the BS can transmit in the downlink part of the frame. The cognitive scheduling strategy is easy to analyze and implement given the following assumptions:

- The BS controls the uplink scheduling for 2-hop links; This true for centralized scheduling of mesh networks.
- Information of node adjacency is available at the BS; Information of R adjacency is always available but $R\,(1+\Delta)$ adjacency is much more difficult to acquire.
- $\Delta \in [0,1]$; Otherwise there is no spatial reuse because if $\Delta \geq 1$ then $\beta_j = 0$ for all $j \in I_m$.
- All network nodes are functional; We consider the total/maximal capacity of the network.
- All stations receive/send data at the same rates; This fair scenario is realistic for commercial use of WiMAX networks when quality of service agreements have been promised to the subscriber. The proposed scheme works for all kind of scheduling strategies, but the conclusions of the analysis could be much different in cases of concentrated traffic in the center of the cell.
- The communication range is smaller than the guard zone, i.e., $\Delta > 0$. As a consequence, only the MS to SN link can be spatially reused.
- All M mesh stations are co-located at distance d from the base station and they use the same sponsor station.

The last assumption is realistic in the context of cell extension in the perimeter of a network. It also corresponds to the worst case scenario for the total capacity of a fair cell. If the MSs have different positions, there will be more room for spatial reuse since the non-overlapping area will be increased, possibly including also the communications by many MS simultaneously. Naturally, this assumption of co-location simplifies the scheduling scheme and makes the problem mathematically more tractable.

It is evident that, out of $N-M-1$ possible co-transmitting SSs, only those that reside outside the intersection of the BS range (disk of radius R) and SN guard zone (disk of radius $R(1+\Delta)$) can be used for spatial reuse. Another constraint is that the MS must reside $R\,(1+\Delta)$ away from the BS in order to avoid interfering with reception at the BS. The non-interfering set A_{SR} is painted grey in Figure 1 and it has area

$$\mathrm{area}(A_{\mathrm{SR}}) = \pi R^2 - \Upsilon\left(r\left(SN, BS\right), R, R\left(1+\Delta\right)\right)$$

with the condition that $d > (1+\Delta)R$.

V. Numerical results

In this section we present numerical results based on the analysis of previous sections and compare them to Monte Carlo simulations. We are using the following parameter setting: $\alpha = 2$, $BW = 1$ bps/Hz, $\Gamma = 7$, $\mathrm{SNR}_0 = 62.4$ dB, $r_0 = 44.37$ meters and $R = 500$ meters.

The analysis of β is beyond the scope of this paper and we consider β only by Monte Carlo simulations. In Figure 2, the estimates of $\mathrm{E}\big[\beta\,|\,R^b \leq R\big]$ are given for $\Delta = 0.2$ and 0.5. These estimates can be inserted to formula (2) while approximating the network capacity.

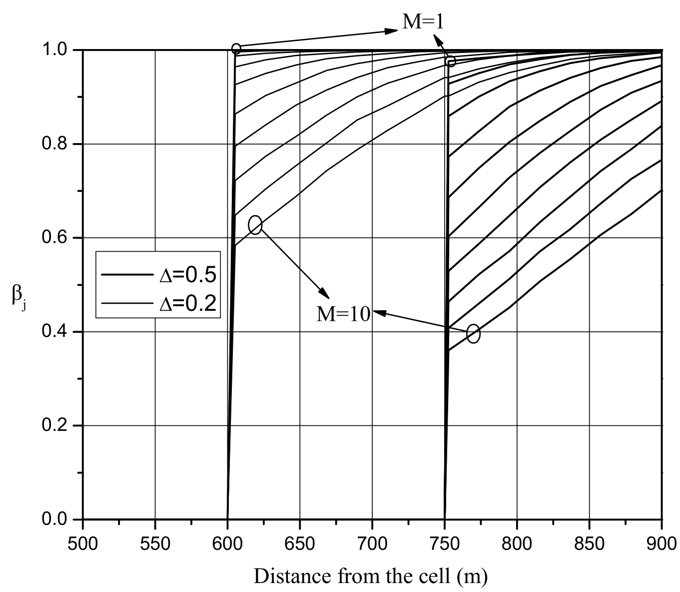

Fig. 2. Estimates of $\mathrm{E}\big[\beta\,|\,R^b \leq R\big]$ for $\Delta = 0.2, 0.5$, $N = 40$ and $M = 1, 2, \ldots, 10$

The mean network capacity in simulations and values of the analytical formula (2) are shown in Figure 3. The results are derived for both the system without spatial reuse ($\beta = 0$) and the system with spatial reuse ($\beta > 0$). The match is surprisingly good, despite the many simplifying assumptions done during the analysis.

In Figure 3, the two basic phenomena of mesh networking are clearly seen. Every node transferred outside the cell area causes a drop in total capacity because of the extra communication links needed. Moreover, as mesh nodes are moving further away from the BS, the capacity of links on the 2-hop path become smaller and the total capacity is again lowered. As discussed already earlier, the condition on $\left\{R^b \leq R\right\}$ is biasing the results because for large d most of the

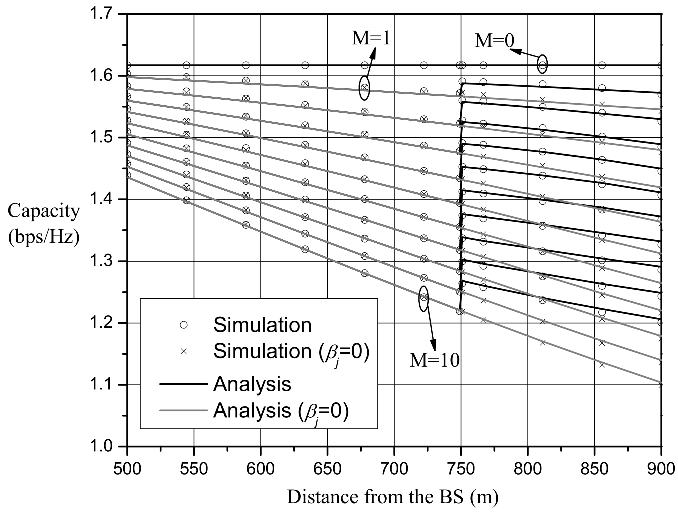

Fig. 3. The mean network capacity for $\Delta = 0.5$, $N = 40$ and $M = 1, 2, \ldots, 10$

unconditioned realizations would contain no potential sponsor station to serve as a relay.

Spatial reuse can be applied when $d > R(1 + \Delta)$ which is demonstrated by the jumps at $d = 750$ meters seen in both figures 2 and 3. The net gain of applying spatial reuse is heavily dependent on the system characteristics and the respective Δ. In our example, where $\Delta = 0.5$, we get an improvement of up to 9.3% in case of $M = 10$ and up to 1.7% when $M = 1$.

The above results show that 2-hop mesh networking can be employed for a reasonable number of customers lying nearby the cell. When the number of mesh customers grows, the capacity of the cell is compromised and the installation of a new BS is advised to cover the extra customers. The positions of out-of-cell customers is also a very important for the final impact of the mesh deployment. Especially, if the mesh nodes are far away, the probability finding a sponsor station becomes very small.

VI. Conclusion and Future Work

This paper presents an analytical approach to the IEEE 802.16 cell capacity in the presence of mesh communications, from the information theory point of view. The optimal, with respect to 2-hop path bottleneck capacity, station located inside the cell is chosen as the sponsor station to provide 2-hop extension of the cell coverage in case of a network perimeter cell. The relative positions of the BS, SSs and MSs as well as the number of stations are important for the performance of the system. Assuming uniform random positions of the stations locating inside the cell, we show how much AMC and spatial reuse can mitigate the capacity loss due to 2-hop paths.

An important issue not studied in this work is whether it would be useful to choose the SNs to be close to the cell boundaries and focus on improved spatial reuse. A related question concerns infrastructure SNs. Given random locations of MSs, what would be the optimal position of an infrastructure SN. Finally, uplink/downlink asymmetry is a common issue in IP networks. It would be of interest to investigate how to balance those two in case of mesh communications for real application scenarios.

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