Multirate Multicast: Optimal Algorithms and Implementation

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***Abstract*—Multirate multicast improves user quality but complicates network optimization. This paper introduces a novel control scheme to dynamically optimize multirate multicast. We present MMT, an adaptive policy which combines differential backlog scheduling and intelligent packet dropping, both based on local information. MMT is shown to maximize network throughput by adapting to changing conditions such as channel quality, network congestion, and device capabilities. Then, we study the problem of per-receiver network utility maximization. To maximize sum utility we propose the MMU policy, an extension of MMT with receiver-end flow control. Under the operation of both policies backlog sizes are deterministically bounded, which provides delay guarantees on delivered packets. An important feature of the proposed scheme is that it does not require source cooperation or centralized calculations. To illustrate its practicality, we present a prototype implementation in the NITOS wireless testbed. Experimental results verify the optimality of the scheme and its low complexity.**

I. Introduction

The increasing demand for multimedia applications, such as real-time conferencing, multiview video and video streaming, pushes data networks to their operational limits and motivates efficient resource allocation schemes. Multicast is a candidate method for delivering multimedia streams to multiple users across a network. To optimize individual user experience, it is desired to employ *multi-rate multicast* transmissions and use layered multimedia coding schemes to adapt users' perceived quality to allowable data rates, see [1], [2]. Since different receivers may require different data rates, we study the problem of *per-receiver Network Utility Maximization* (NUM) in multi-rate multicast, where each receiver is assigned a potentially different utility function.

Controlling multicast streams is challenging; the optimal network resource allocation and stream rate adaptation depends on the network state, which includes channel quality, network congestion, user demand and device capabilities. Current approaches collect network state information at the source and calculate the achievable stream rate per receiver. Such a process can be overwhelming for the source, since a multicast stream may have thousands of receivers. In this work we develop a solution to per-receiver NUM in multi-rate multicast without source cooperation.

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Our solution combines scheduling with intelligent packet dropping at intermediate nodes. Packets corresponding to all stream layers are initially injected into the network without any calculations. Progressively, some packets are dropped according to a dropping scheme which bases its decisions on local information. We show that the original stream is stripped of unnecessary packets so that each receiver obtains the exact amount of information that corresponds to maximum throughput. Moreover, we combine the above mechanism with receiver-end flow control to produce a scheme that maximizes utility without source cooperation.

The proposed policies have the following attractive features. *First*, they dynamically track the optimal solution without explicitly exchanging information about time-varying system parameters such as stream rate, link quality and network congestion. *Second*, they are based on neighbor backlog information which is found locally. Thus the policies are amenable to distributed implementation for wireless and heterogeneous network technologies. *Third*, they do not require source cooperation, i.e. the sources transmit stream packets without calculating the achievable receiver rates–this simplifies multi-rate multicast for networks with a large number of receivers. *Last*, they yield deterministic bounds for the queue backlogs, which provides delay guarantees and facilitates implementation on systems with finite buffers. Our contribution is summarized in the following points:

- We present the Maximum Multicast Throughput (MMT) policy, which achieves near optimal throughput for multi-rate multicast. MMT uses backpressure-type scheduling and threshold-based packet dropping.
- We develop the Maximum Multicast Utility (MMU) policy, which additionally includes a utility-based flow controller at the receivers. MMU is shown to solve the per-receiver NUM problem.
- We propose MMU-W, a heuristic modification for operation on IEEE 802.11-based wireless devices. We implement MMU-W in a wireless testbed and perform experiments. The results demonstrate the optimality and the low complexity of our scheme.

A. Related Work

The problem of NUM has been extensively studied for the case of unicast sessions [3]. For multicast sessions, [4] provides a fair-utility offline solution. Optimizing resource allocation and maximizing network utility by solving offline

optimization problems is less desirable since variability in the network renders this approach ineffective. Every change requires re-solving the problem and enforcing new rules. A preferable alternative is a dynamic policy that achieves the long-term goals by making adaptive real-time decisions [5].

In [6], an approach to dynamic control for multicast sessions balances the stream across a selection of multicast trees. While this approach provides maximum throughput, it requires the arrivals to be feasible, which can only be achieved with proper flow control. Many existing flow control approaches have the users estimate their maximally allowable throughput and convey this information to the source, which then creates a virtual multicast session for each stream layer [7], [8]. Recently, [9] proposed a dynamic policy that solves the stochastic NUM problem under the assumption of infinite demand, using the concept of the *shadow backpressure* routing. Virtual packets travel in the reverse direction in order to discover congestion and help route the actual data packets.

Our approach differs because it is based on local dropping and it does not require end-to-end signaling. An in-network flow control approach is proposed in [10], where a credit-based flow controller is shown to achieve *max-min* fairness, i.e. it solves the NUM problem for a specific choice of utility functions. We generalize in-network flow control for multirate multicast per-receiver NUM.

II. System Model

Let $G = (V, E)$ be the graph, serving a set C of multicast sessions. Session $c \in C$ consists of a source node $c \in V$ [1] and a set of receivers $U^{(c)}$, and is delivered over a given tree $G^{(c)} = (V^{(c)}, E^{(c)}) \subset G$. We consider a wireline network in which all links in E can be used simultaneously (we discuss wireless networks in section V).

Time is slotted and in slot t, $A^{(c)}(t)$ packets arrive at the source of session c. We assume that $A^{(c)}(t)$ are i.i.d. over time slots with mean $\lambda^{(c)}$ and take finite values, i.e. $A^{(c)}(t) \leq A_{\max}$.

A. Queueing Structure

Each node maintains one *transmission queue* for every outgoing link l and session c, let $Q_l^{(c)}(t)$ denote its backlog at the beginning of slot t. Define $p(l) \in E^{(c)}$ to be the incoming (parent) link to that node[2] and let $L_{\text{out}}(c) \subset E^{(c)}$ be the set of outgoing links of the source node c. Queue $Q_l^{(c)}(t)$ evolves across slots according to

$$Q_l^{(c)}(t+1) \leq \left[\left(Q_l^{(c)}(t) - \mu_l^{(c)}(t)\right)^+ - d_l^{(c)}(t)\right]^+ + A^{(c)}(t)1_{[l\in L_{\text{out}}(c)]} + \mu_{p(l)}^{(c)}(t),\ l \in E^{(c)}, \quad (1)$$

where $\mu_l^{(c)}(t)$ is the allocated transmission rate and $d_l^{(c)}(t)$ is the number of packets that are dropped from $Q_l^{(c)}(t)$. Let $\mu_l^{\max}$ denote the capacity of link l. The capacity constraint $\sum_{c\in C} \mu_l^{(c)}(t) \leq \mu_l^{\max}$ must be satisfied in every slot. Also, we impose $d_l^{(c)}(t) \in [0, d_{\max}]$, where $d_{\max}$ is a system-defined parameter. Throughout the paper, we assume $d_{\max} \geq A_{\max}+\mu_{\max}$, where $\mu_{\max} \triangleq \max_{l\in E} \mu_l^{\max}$ is the maximum link capacity. The value $A_{\max} + \mu_{\max}$ is an upper bound to the incoming data rate to a node, and our choice of $d_{\max}$ ensures that the packet dropping rate is large enough so that all transmission queues can always be stabilized.

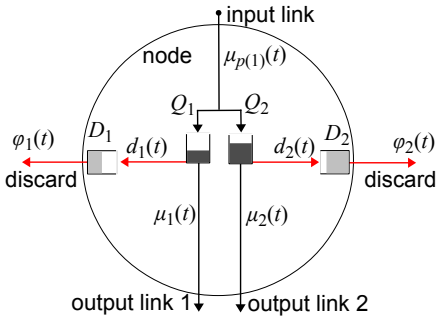

Fig. 1. The proposed queue structure on an node with one incoming and two outgoing links (we show one session and we omit the session notation). Each link $l = 1, 2$ is associated with a transmission queue $Q_l(t)$ and a dropping queue $D_l(t)$.

Let $\widetilde{d}_l^{(c)}(t) = \min[Q_l^{(c)}(t), d_l^{(c)}(t)]$ be the actual packets dropped from $Q_l^{(c)}(t)$ in slot t, which can be smaller than $d_l^{(c)}(t)$ if there are not enough packets to be dropped. To provide direct control over the amount of dropped packets, we set up a *drop queue* $D_l^{(c)}(t)$ associated with each transmission queue $Q_l^{(c)}(t)$. Before leaving the system, the dropped packets are "moved" to the drop queue from which they are later discarded according to the control policy. The drop queue $D_l^{(c)}(t)$ evolves across slots according to

$$D_l^{(c)}(t+1) = \left[D_l^{(c)}(t) - \varphi_l^{(c)}(t)\right]^+ + \widetilde{d}_l^{(c)}(t),\ l \in E^{(c)}, \quad (2)$$

where $\varphi_l^{(c)}(t) \in [0, d_{\max}]$ is a decision variable that determines the number of packets that are eventually removed from the network in slot t. Note that the packets in drop queues $D_l^{(c)}(t)$ are not going to be transmitted, and therefore it practically suffices to keep track of the values of $D_l^{(c)}(t)$ only as counters.

Focusing on a network node, our queueing mechanism works as follows. All arriving packets are replicated to each transmission queue $Q_l^{(c)}(t)$, for example see Fig. 1. In a slot t, $\mu_l^{(c)}(t)$ determines the number of session c packets transmitted on link l, $d_l^{(c)}(t)$ decides the number of packets that are internally moved from $Q_l^{(c)}(t)$ to the drop queue $D_l^{(c)}(t)$, and $\varphi_l^{(c)}(t)$ is the number of packets that are discarded from queue $D_l^{(c)}(t)$ and permanently removed from the network. A control policy chooses the values of the decision variables, $\mu_l^{(c)}(t)$, $d_l^{(c)}(t)$ and $\varphi_l^{(c)}(t)$ at each slot.

III. Throughput Maximization

In order to introduce the concepts and the notation, we begin by considering the problem of maximizing the sum throughput of all receivers in multiple multicast sessions. In Section IV,

[1]To simplify the notation we do not allow different sessions to have the same source. This limitation can be waived without affecting the results.

[2]Only one such incoming link $p(l)$ exists since $G^{(c)}$ is a tree. Note that the value of $p(l)$ depends on the multicast session under consideration, and we abuse the notation to simplify exposition.

we study the more general problem of utility maximization.

A. Flow-Level Characterization

Before we develop the dynamic network control policy, it is useful to provide a flow-level characterization of the optimal throughput vector and the optimal packet dropping rates, presented as solutions to linear optimization problems. These flow-level solutions are useful for proving the optimality of our control policies. However, the proposed policies solve these problems in a distributed and dynamic manner without the need to explicitly compute these solutions.

We define $f_l^{(c)}$ to be the average session c data flow rate over link l and $q_l^{(c)}$ the average packet dropping rate at queue $Q_l^{(c)}$. These flow variables must satisfy the flow conservation and link capacity constraints:

$$\lambda^{(c)} = f_l^{(c)} + q_l^{(c)}, \quad l \in L_{\text{out}}(c), \ \forall c \tag{3}$$

$$f_{p(l)}^{(c)} = f_l^{(c)} + q_l^{(c)}, \quad l \in E^{(c)} \setminus L_{\text{out}}(c), \ \forall c \tag{4}$$

$$\sum_{c \in C} f_l^{(c)} \le \mu_l^{\max}, \ f_l^{(c)} = 0 \text{ if } l \notin G^{(c)}, \quad l \in E, \tag{5}$$

The packet dropping rate vector $(q_l^{(c)})$ is said to be feasible if there exist flow variables $f_l^{(c)}$ that satisfy (3)-(5).

Let $r_u^{(c)}$ denote the throughput of receiver $u \in U^{(c)}$. Let Λ be the set of feasible throughput vectors $(r_u^{(c)})$. We have

$$\Lambda = \left\{ (r_u^{(c)}) \;\middle|\; \begin{array}{c} \text{(3)-(5) hold, } r_u^{(c)} = f_{l_u}^{(c)} \\ f_l^{(c)}, q_l^{(c)} \ge 0 \end{array} \right\}, \tag{6}$$

where l_u is the incoming link of the receiver u in session c. In (6), $r_u^{(c)} = f_{l_u}^{(c)}$ states that the throughput of a receiver is equal to its incoming flow rate.

The problem of maximizing the sum throughput of all receivers in the network is

$$\text{maximize} \ \sum_{c,u} r_u^{(c)}, \quad \text{subject to } (r_u^{(c)}) \in \Lambda. \tag{7}$$

It is useful to consider an equivalent optimization problem that minimizes packet dropping rates. Let $E_u^{(c)}$ denote the set of links that form the path from the source node c to a receiver u. Summing (4) over $l \in E_u^{(c)}$ and using $r_u^{(c)} = f_{l_u}^{(c)}$, we have

$$r_u^{(c)} = \lambda^{(c)} - \sum_{l \in E_u^{(c)}} q_l^{(c)}, \tag{8}$$

which states that the throughput $r_u^{(c)}$ of receiver u is equal to the exogenous data arrival rate less the sum of packet dropping rates along the path $E_u^{(c)}$ to u. Summing (8) over all receivers $u \in U^{(c)}$ in a session, the total session c throughput is

$$\sum_{u \in U^{(c)}} r_u^{(c)} = \left| U^{(c)} \right| \lambda^{(c)} - \sum_{l \in E} m_l^{(c)} \, q_l^{(c)}, \tag{9}$$

where $m_l^{(c)}$ is the number of session c receivers connected to their source via link l. From (9) we see that maximizing the total throughput of session c is equivalent to minimizing the weighted packet dropping rate $\sum_{l \in E} m_l^{(c)} \, q_l^{(c)}$. Consequently, the throughput maximization problem (7) is equivalent to the minimization problem

$$\text{minimize} \ \sum_{c,l} m_l^{(c)} q_l^{(c)}, \quad \text{subject to } (q_l^{(c)}) \text{ feasible.} \tag{10}$$

Next, we design a control policy that stabilizes all queues in the network and achieves optimal packet dropping rates; from the equivalence of (7) and (10), our policy achieves the maximum total throughput as well.

B. Intuition for Packet-Level Control

To measure the degree of congestion in the network, we construct a strictly increasing function of the queue backlogs $Q_l^{(c)}(t)$ and $D_l^{(c)}(t)$, i.e., we define the weighted quadratic Lyapunov function

$$L(t) = \frac{1}{2} \sum_c \sum_{l \in E} m_l^{(c)} ([Q_l^{(c)}(t)]^2 + [D_l^{(c)}(t)]^2).$$

The quadratic terms are weighted by $m_l^{(c)}$ because the importance of a queue is proportional to the number of receivers connected to that queue. Let $H(t) = (Q_l^{(c)}(t); D_l^{(c)}(t))$ be the queue backlog vector in slot t. Define the Lyapunov drift

$$\Delta(t) = \mathbb{E}\left[L(t+1) - L(t) \mid H(t) \right] \tag{11}$$

as the expected difference of the congestion measure $L(t)$ over a slot. A control policy that minimizes the Lyapunov drift in every slot suffices to stabilize the network and keep the queue backlogs bounded [5].

Recall from (10), that we also seek to minimize the weighted time-average packet dropping rate

$$\sum_{c,l} m_l^{(c)} q_l^{(c)} = \sum_{c,l} m_l^{(c)} \overline{d}_l^{(c)} \triangleq \sum_{c,l} m_l^{(c)} \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\widetilde{d}_l^{(c)}(\tau)]. \tag{12}$$

If a drop queue $D_l^{(c)}(t)$ is stable, then from queueing theory its arrival rate must be less than or equal to its time-average service rate, i.e., from (2) we have

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\widetilde{d}_l^{(c)}(\tau)] \le \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\varphi_l^{(c)}(\tau)]. \tag{13}$$

Our approach forces this bound to be tight, and hence minimizing (12) can be achieved by minimizing its upper bound in (13), provided all $D_l^{(c)}(t)$ queues are stable. In fact, it suffices to minimize in every slot the sum $\sum_{c,l} m_l^{(c)} \mathbb{E}_H[\varphi_l^{(c)}(t)]$, where $\mathbb{E}_H[\cdot]$ is a compact notation for the conditional expectation $\mathbb{E}[\cdot \mid H(t)]$.

Minimizing both the Lyapunov drift $\Delta(t)$ and the sum $\sum_{c,l} m_l^{(c)} \mathbb{E}_H[\varphi_l^{(c)}(t)]$ induces a conflict, because the network becomes more congested when less packets are dropped. It is therefore natural to consider minimizing a weighted sum of these two metrics,

$$\Delta(t) + V \sum_{c,l} m_l^{(c)} \, \mathbb{E}_H[\varphi_l^{(c)}(t)], \tag{14}$$

where $V > 0$ is predefined parameter that reflects the relative importance of minimum packet dropping to queue stability. As we will see, V also controls a tradeoff between the performance gap of our policy from optimality and the required finite buffer size in the transmission queues $Q_l^{(c)}(t)$.

C. The Proposed Policy

Our policy arises from the minimization of (14). Performing standard calculation of the Lyapunov drift we obtain a bound

$$\begin{aligned}(14) \le B_1 + \underbrace{\sum_{c,l\in L_{\text{out}}(c)} m_l^{(c)} Q_l^{(c)}(t)\,\lambda^{(c)}}_{\text{constant}} \\ \underbrace{-\sum_{c,l} m_l^{(c)} \mathbb{E}_H[d_l^{(c)}(t)]\big(Q_l^{(c)}(t) - D_l^{(c)}(t)\big)}_{\text{dropping}} \\ \underbrace{-\sum_{c,l} m_l^{(c)} \mathbb{E}_H[\varphi_l^{(c)}]\big(D_l^{(c)}(t) - V\big)}_{\text{discarding}} \\ \underbrace{-\sum_{c,l} \mathbb{E}_H[\mu_l^{(c)}(t)] W_l^{(c)}(t)}_{\text{scheduling}} \qquad (15)\end{aligned}$$

where $B_1 > 0$ is a finite constant given in the report [11] and

$$W_l^{(c)}(t) \triangleq m_l^{(c)} Q_l^{(c)}(t) - \sum_{l':p(l')=l} m_{l'}^{(c)} Q_{l'}^{(c)}(t) \qquad (16)$$

is the weighted differential backlog. Fig. 2 gives an example calculation of $W_l^{(c)}(t)$. Next, we propose a throughput-optimal policy that is designed to minimize the RHS of (15) at each slot.

Maximum Multicast Throughput (MMT) Policy

Packet Dropping: Each transmission queue $Q_l^{(c)}(t)$ moves $\min\{d_l^{(c)}(t), Q_l^{(c)}(t)\}$ packets to its drop queue $D_l^{(c)}(t)$ at the end of slot t, where

$$d_l^{(c)}(t) = \begin{cases} d_{\max} & \text{if } Q_l^{(c)}(t) > D_l^{(c)}(t) \\ 0 & \text{otherwise.} \end{cases} \qquad (17)$$

The drop queue $D_l^{(c)}(t)$ removes $\min\{\varphi_l^{(c)}(t), D_l^{(c)}(t)\}$ packets from the network according to

$$\varphi_l^{(c)}(t) = \begin{cases} d_{\max} & \text{if } D_l^{(c)}(t) > V \\ 0 & \text{otherwise.} \end{cases} \qquad (18)$$

Scheduling: Let C_l be the set of multicast sessions that use link l. Define $W_l^*(t) = \max_{c\in C_l} W_l^{(c)}(t)$ and let c_l^* be a maximizer session (ties are broken arbitrarily). We allocate the link rate

$$\mu_l^{(c_l^*)}(t) = \begin{cases} \mu_l^{\max} & \text{if } W_l^*(t) > 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (19)$$

Let $\mu_l^{(c)}(t) = 0$ for all the other sessions $c \in C_l \setminus \{c_l^*\}$.

Observe that (17) minimizes the dropping term of (15), (18) minimizes the discarding term and (19) minimizes the scheduling term. Since the first two terms in (15) are constant, we conclude that MMT minimizes the RHS of (15).

We note that the policy operates in a distributed manner using only locally available information. For the computation of $W_l^{(c)}(t)$, we require knowledge of the neighbor backlogs.

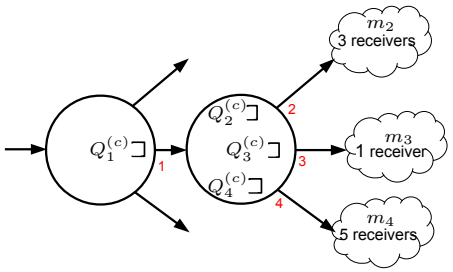

Fig. 2. Illustration of the differential backlog calculation in MMT policy; $W_1^{(c)} = 9Q_1^{(c)} - 3Q_2^{(c)} - Q_3^{(c)} - 5Q_4^{(c)}$.

As shown in prior work, this is not restrictive for practical applications, e.g. see [12]. Also, delayed backlog information is sufficient for throughput optimality, see [13, §4.7].

D. Performance Evaluation of MMT

Due to the dropping mechanism in (17)-(18), $Q_l^{(c)}(t)$ and $D_l^{(c)}(t)$ are deterministically bounded. Applying the approach of [14] we have the following result.

Lemma 1. All queues $Q_l^{(c)}(t)$ and $D_l^{(c)}(t)$ are deterministically bounded by

$$Q_l^{(c)}(t) \le V + 2d_{\max},\ D_l^{(c)}(t) \le V + d_{\max},\ \forall l, c, t.$$

Hence, a buffer size of $V+2d_{\max}$ is sufficient to avoid unexpected queue overflow at $Q_l^{(c)}(t)$. The MMT policy achieves near-optimal total throughput as the following theorem asserts.

Theorem 1 (Optimality of MMT)**.** The MMT policy yields the total throughput satisfying

$$\sum_{c,u} \overline{r}_u^{(c)} \ge \sum_{c,u} r_u^{(c)*} - \frac{B_1}{V}.$$

Where

$$\overline{r}_u^{(c)} \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left[\widetilde{\mu}_{l_u}^{(c)}(t)\right]$$

is the throughput of receiver u in multicast session c and $(r_u^{(c)*})$ is a solution to (7). The performance gap B_1/V can be made arbitrarily small by choosing a sufficiently large $V > 0$. The technical report [11] includes the proof of Theorem 1.

E. Simulation of MMT

We illustrate how MMT adapts to changing conditions via simulations. Consider the multicast scenario of Fig. 3. Two multicast sessions share link (a, b). The set of multicast receivers are $U^{(1)} = \{\mathrm{b}, \mathrm{c}\}$ for session 1 and $U^{(2)} = \{\mathrm{b}, \mathrm{d}, \mathrm{e}\}$ for session 2. Links (b, d) and (b, e) have capacity x, while the rest links have unit capacities, i.e. 1packet/slot. Both sessions have unit arrival rate. We seek to maximize total throughput.

Observe that throughput maximization depends crucially on the value of x. For example, if $x = 1$, then maximum throughput is achieved by allocating all the resources of link (a, b) to session 2, since session 2 has three receivers and session 1 has two. If on the other hand $x = 0$, then maximum throughput is achieved by allocating all the resources of link (a, b) to session 1. In general, for $x \in [0, 1]$, throughput is maximized if the allocation on link (a, b) is x to session 2

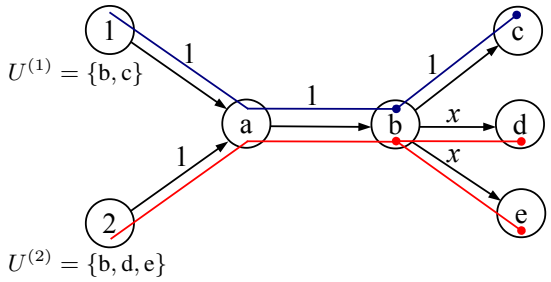

Fig. 3. An example of multirate multicast with two sessions. Session 1 uses the link set $E^{(1)} = \{(1, a), (a, b), (b, c)\}$ and session 2 uses the link set $E^{(2)} = \{(2, a), (a, b), (b, d), (b, e)\}$. The set of receivers are denoted with $U^{(1)}, U^{(2)}$. Numbers on links indicate capacities.

and $1-x$ to session 1. Note, that the packet dropping decision of node $\{a\}$ depends on the quality of links (b, d) and (b, e), information which is not directly available at $\{a\}$.

In the simulation we vary the value x. Initially $x = 1$ and gradually x reduces in steps of 0.1. Fig. 4 (left) shows the receiver c throughput. According to the above discussion, the optimal average throughput is equal to $1 - x$, showcased in the Figure with gray line. The simulations showed that the average throughput of MMT is equal to $1-x$. Hence, we show the instantaneous throughput averaged in moving windows of 100 slots. At each interval, the throughput converges quickly to the optimal, which shows how MMT adapts to changing conditions.

In Fig. 4 (right), we showcase the backlog at node b with packets destined to node c, for the same sample path. In this simulation we have used $V = 25$ and $d_{\max} = 5$ and by Lemma 1, the backlog is upper bounded by 35 packets. In the simulations, the backlog never exceeds 26 packets despite the link quality variations and the randomness of the arrivals.

IV. Utility Maximization

Next we consider the per-receiver NUM problem. Solving this general problem allows to use different utility functions to achieve several objectives such as maximum throughput (studied separately in the previous section), $\alpha-$fairness which includes proportional and max-min fairness as special cases, user priority, and satisfying user-specific quality requirements.

A. Per-Receiver NUM Problem Formulation

In multicast session c, a receiver u has a utility function $g_u^{(c)}$, which is assumed to be concave, increasing and continuously differentiable with bounded derivatives.[3] Consider the per-receiver NUM problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{c,u} g_u^{(c)}(r_u^{(c)}) \\ \text{subject to} \quad & (r_u^{(c)}) \in \Lambda. \end{aligned} \tag{20}$$

Define the auxiliary function

$$h_u^{(c)}(x) \triangleq g_u^{(c)}(x) - \theta x,$$

[3]We assume $[g_u^{(c)}]'(x) \leq [g_u^{(c)}]'(0) < \infty$. Utility functions that have unbounded derivatives as $x \to 0$, such as $\log(x)$, can be approximated by those with bounded derivatives. For example, we can approximate $\log(x)$ by $\log(x + \xi)$ for some small $\xi > 0$.

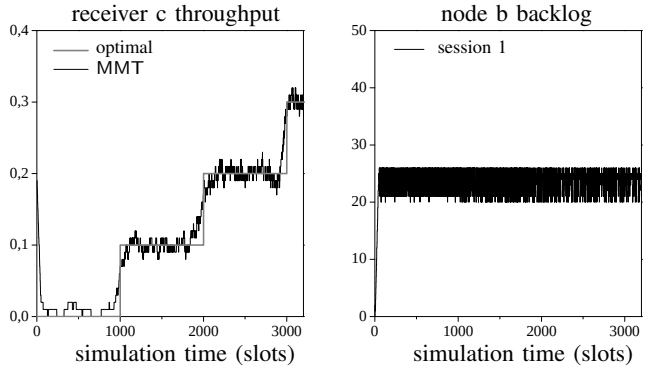

Fig. 4. Performance when varying quality for links $(b, d), (b, e)$ in the topology of Fig. 3. The left Figure compares MMT to the optimal average throughput of receiver c. The right Figure shows the backlog of node b with packets for transmission to receiver c.

where $\theta > 0$ is a parameter decided later. Then, maximizing the total utility $\sum_{c,u} g_u^{(c)}(r_u^{(c)})$ is equivalent to maximizing

$$\begin{aligned} &\sum_{c,u} \left(h_u^{(c)}(r_u^{(c)}) + \theta\, r_u^{(c)}\right) \\ &\quad = \sum_{c,u} h_u^{(c)}(r_u^{(c)}) + \theta \sum_{c,u} \left(\lambda^{(c)} - \sum_{l \in E_u^{(c)}} q_l^{(c)}\right) \\ &\quad = \sum_{c,u} h_u^{(c)}(r_u^{(c)}) - \theta \sum_{c,l} m_l^{(c)} q_l^{(c)} + \theta \sum_{c,u} \lambda^{(c)}, \end{aligned} \tag{21}$$

where the last sum is an (unknown) constant. In what follows, we modify our mechanism so that by controlling functions $\mu_l^{(c)}(t), d_l^{(c)}(t), \varphi_l^{(c)}(t)$ and a new virtual queue which will introduce next, the system is driven to the solution of (21).

B. Receiver virtual queue $Z_u^{(c)}(t)$

At each multicast receiver u, we set up the virtual queue $Z_u^{(c)}(t)$, which tracks the deficit/surplus of session c packets received at that user and evolves as

$$Z_u^{(c)}(t+1) = [Z_u^{(c)}(t) - \nu_u^{(c)}(t)]^+ + \widetilde{\mu}_{l_u}^{(c)}(t),$$

where $l_u \in E_u^{(c)}$ is the incoming link of node u, and

$$\widetilde{\mu}_{l_u}^{(c)}(t) = \min\left[Q_{l_u}^{(c)}(t), \mu_{l_u}^{(c)}(t)\right]$$

is the actual number of packets delivered to that user. The departures $\nu_u^{(c)}(t)$ are controlled by the policy and chosen in the interval $[0, \nu_{\max}]$, we choose $\nu_{\max}$ below. The functionality of this virtual queue is to track the urgency of a receiver to obtain more packets: if $Z_u^{(c)}(t)$ is small, receiver u must urgently obtain packets for the maximum utility to be preserved.

We also define the *virtual pressure* for each receiver u which is regulated by the virtual queue:

$$Y_u^{(c)}(t) \triangleq \begin{cases} we^{w(Z_u^{(c)}(t) - \zeta)}, & \text{if } Z_u^{(c)}(t) \geq \zeta, \\ -we^{w(\zeta - Z_u^{(c)}(t))}, & \text{otherwise,} \end{cases} \tag{22}$$

where w, ζ are positive parameters whose value will be chosen later. Note, that in normal backpressure, the pressure of a destination node is zero, while in our policy $Y_u^{(c)}(t)$ can take positive or even negative values. The sign of $Y_u^{(c)}(t)$ indicates the urgency of the particular receiver to obtain more or less packets according to the requested objective. Indeed, the behavior of $Y_u^{(c)}(t)$ is controlled by $\nu_u^{(c)}(t)$, which as we

will see shortly, is chosen according to the utility function.

C. The Proposed Policy

Let $H(t) = (Q_l^{(c)}(t); D_l^{(c)}(t); Z_u^{(c)}(t))$ be the joint queue backlog vector in slot t. Define the Lyapunov function

$$L(t) = \frac{1}{2}\sum_{c,l} m_l^{(c)}\Big([Q_l^{(c)}(t)]^2 + [D_l^{(c)}(t)]^2\Big) + \frac{1}{2}\sum_{c,u}\Big(e^{w(Z_u^{(c)}(t)-\zeta)} + e^{w(\zeta - Z_u^{(c)}(t))}\Big).$$

Note, that the Lyapunov function is composed of two terms, the quadratic term is identical to the Lyapunov function used in throughput maximization section, while the exponential term is identical to the one used for receiver-based flow control for unicast sessions in [15].

Recall the definition of Lyapunov drift $\Delta(t)$ from (11). In order to solve the problem in (21) we define the weighted objective:

$$\Delta(t) + V\left[\theta\sum_{c,l} m_l^{(c)}\mathbb{E}_H\big[\varphi_l^{(c)}(t)\big] - \sum_{c,u}\mathbb{E}_H\big[h_u^{(c)}(\nu_u^{(c)}(t))\big]\right]. \tag{23}$$

Using standard drift derivation techniques we obtain the following bound

$$\begin{aligned}(23) \le\ & \underbrace{B_2 + \sum_{c,l\in L_{\text{out}}(c)} m_l^{(c)}Q_l^{(c)}(t)\,\lambda^{(c)} + \frac{\epsilon}{2}\sum_{uc} Y_u^{(c)}(t)}_{\text{(constant)}} \\ & \underbrace{-\sum_{c,l} m_l^{(c)}\mathbb{E}_H[d_l^{(c)}(t)](Q_l^{(c)}(t) - D_l^{(c)}(t))}_{\text{(dropping)}} \\ & \underbrace{-\sum_{c,l} m_l^{(c)}\mathbb{E}_H[\varphi_l^{(c)}(t)](D_l^{(c)}(t) - V\theta)}_{\text{(discarding)}} \\ & \underbrace{-\sum_{uc}\mathbb{E}_H\left\{Vh_u^{(c)}(\nu_u^{(c)}(t)) + Y_u^{(c)}(t)\nu_u^{(c)}(t)\right\}}_{\text{(flow control)}} \underbrace{-\ \Psi(t)}_{\text{(scheduling)}}\end{aligned} \tag{24}$$

where $\epsilon > 0$ is a parameter, B_2 is a large constant defined in [11] and

$$\Psi(t) \triangleq -\sum_{uc}\mathbb{E}_H[\mu_{l_u}^{(c)}(t)]\,Y_u^{(c)}(t) + \sum_{c,l} m_l^{(c)}Q_l^{(c)}(t)\,\mathbb{E}_H\big[\mu_l^{(c)}(t) - \mu_{p(l)}^{(c)}(t)\big].$$

Let $1_{[l,u]}^{(c)}$ be the indicator function on the event that the tail node of l on $G^{(c)}$ is a receiver $u \in U^{(c)}$. Then, define the weighted differential backlog as

$$W_l^{(c)}(t) = m_l^{(c)}Q_l^{(c)}(t) - \sum_{l':\,p(l')=l} m_{l'}^{(c)}Q_{l'}^{(c)}(t) - 1_{[l,u]}^{(c)}Y_u^{(c)}(t). \tag{25}$$

Observe that the virtual pressure $Y_u^{(c)}(t)$ is applied only if the tail node of l is a receiver for this session. By rearranging terms, we have

$$\Psi(t) = \sum_{c,l}\mathbb{E}_H[\mu_l^{(c)}(t)]\,W_l^{(c)}(t).$$

We design our Maximum Multicast Utility (MMU) policy to minimize the RHS of (24). To achieve this, we add a receiver-end flow controller, similar to the one used in [15].

TABLE I
PARAMETER SELECTION GUIDELINES.

Parameter	Explanation	Suggested values
$m_l^{(c)}$	number of session c users connected to source c through link l	problem defined
$\mu_l^{\max}$	capacity of link l	problem defined
$\mu_{\max}$	maximum link capacity	$\max_l \mu_l^{\max}$
$d_{\max}$	drop batch size	$d_{\max} \ge A_{\max} + \mu_{\max}$
ϵ	utility gap parameter	> 0
$\nu_{\max}$	maximum value for $\nu_u^{(c)}(t)$	$\mu_{\max} + \epsilon/2$
$\delta_{\max}$	bound on $\lvert\nu_u^{(c)}(t) - \mu_{l_u}^{(c)}(t)\rvert$	$\max[\nu_{\max}, \mu_{\max}]$
w	multiplier in (22)	$\frac{\epsilon}{\delta_{\max}^2}e^{-\epsilon/\delta_{\max}}$
ζ	central value for $Z_u^{(c)}(t)$	$\ge \nu_{\max}$
$g_u^{(c)}(x)$	user utility function	objective specific
θ	upper bound on $[g_u^{(c)}]'(x)$, $x \ge \epsilon$	$\max_{u,c}[g_u^{(c)}]'(0)$
$h_u^{(c)}(x)$	auxiliary function	$g_u^{(c)}(x) - \theta x$
V	utility gap/backlog size tradeoff	$V\theta + 2d_{\max} \ge w$

We design our Maximum Multicast Utility (MMU) policy to minimize the RHS of (24). To achieve this, we add a receiver-end flow controller, similar to the one used in [15].

Maximum Multicast Utility (MMU) Policy

Parameter Selection: Choose positive parameters V, $d_{\max}$, $\nu_{\max}$, w, ζ, and θ as summarized in the Table I. For a discussion on these parameter choices see [14, §V-C]. Initialize the queues with $Q_l^{(c)}(0) = 0$, $Z_u^{(c)}(0) = \zeta + \frac{1}{w}\log\left(\frac{V\theta}{w}\right)$ and $D_l^{(c)}(0) = V\theta$.

Packet Dropping: Same as in MMT policy.

Receiver-End Flow Control: Choose $\nu_u^{(c)}(t)$ to be the solution to

$$\text{maximize} \quad Vh_u^{(c)}(x) + Y_u^{(c)}(t)\,x \tag{26}$$
$$\text{subject to} \quad 0 \le x \le \nu_{\max}, \tag{27}$$

where $Y_u^{(c)}(t)$ is given in (22).

Scheduling: Same as in MMT policy, except that we use (25) as the value of $W_l^{(c)}(t)$, instead of (16).

D. Performance Evaluation of MMU

Lemma 2. Under the MMU policy, all queues $Q_l^{(c)}(t)$, $D_l^{(c)}(t)$, and $Z_u^{(c)}(t)$ are deterministically bounded by

$$Q_l^{(c)}(t) \le V\theta + 2d_{\max},\ D_l^{(c)}(t) \le V\theta + d_{\max},\ \forall c,l,t,$$
$$Z_u^{(c)}(t) \le \zeta + \frac{1}{w}\log\left(\frac{V\theta + 2d_{\max}}{w}\right) + \mu_{\max},\ \forall c,u,t.$$

Theorem 2 (Optimality of MMU)**.** The MMU policy achieves the long-term utility satisfying

$$\sum_{c,u} g_u^{(c)}(\overline{r}_u^{(c)}) \ge \sum_{uc} g_u^{(c)}(r_u^{(c)*}) - \frac{B_2}{V} - \frac{3\epsilon}{2}\sum_{c,u}\big([g_u^{(c)}]'(0) + \theta\big), \tag{28}$$

where $(r_u^{(c)*})$ is the utility-optimal throughput vector.

E. Achieving Throughput Requirements

We show how to use the MMU policy to deliver a video stream to users with strict throughput requirements. Consider

TABLE II
MMU SIMULATION RESULTS FOR PRIORITIZING BASE LAYER PACKETS.

	Session 1		Session 2		
receivers	b	c	b	d	e
stream rate	0.996		0.998		
$\xi_u^{(c)}$	0.2	0.2	0.2	0.2	0.2
$\bar{r}_u^{(c)}$	0.1948	0.1948	0.805	0.805	0.805

base layer packets breakdown					
stream rate	0.1997		0.199		
received rate	0.1944	0.1944	0.199	0.199	0.199
delivery ratio	97.35%	97.35%	100%	100%	100%

enhancement layer packets breakdown					
stream rate	0.7963		0.799		
received rate	0.0003	0.0003	0.606	0.606	0.606
delivery ratio	0.037%	0.037%	75.84%	75.84%	75.84%

the optimization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{c,u\in U^{(c)}} g_u^{(c)}(r_u^{(c)}) \qquad (29)\\ \text{subject to} \quad & (r_u^{(c)}) \in \Lambda, \\ & (r_u^{(c)}) \geq (\xi_u^{(c)}), \end{aligned}$$

where the inequality is element-wise and $\xi_u^{(c)}$ denotes the throughput requirement of session c receiver u. We assume problem (29) admits a feasible solution. In order to solve (29) using MMU, we use the penalty method, see [16, §4.7]. Define the penalty function

$$\pi\big[(x_u^{(c)})\big] \triangleq K \sum_{c,u\in U^{(c)}} \left(\xi_u^{(c)} - x_u^{(c)}\right)^+,$$

where $(x_u^{(c)})$ is a vector with one element for every receiver-session pair. If all requirements are satisfied (i.e. $r_u^{(c)} \geq \xi_u^{(c)}$, $\forall u$) then $\pi\big[(r_u^{(c)})\big] = 0$. If some requirement is violated, then $\pi\big[(r_u^{(c)})\big]$ increases proportionally to K and to the norm-1 distance of $(r_u^{(c)})$ from the feasible set. Also note that π is convex and thus $-\pi$ is concave. Next, consider a convex optimization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{c,u\in\mathcal{U}^{(c)}} \left[g_u^{(c)}(r_u^{(c)}) - K\left(\xi_u^{(c)} - r_u^{(c)}\right)^+ \right] \qquad (30)\\ \text{subject to} \quad & (r_u^{(c)}) \in \Lambda, \end{aligned}$$

By letting $K \to \infty$, the solution of (30) converges to the solution of (29) [16]. A practical approach is to pick a "large" finite value for K.

F. Simulations: Prioritizing Base Layer Packets

In multimedia streaming with layer coding, the stream reconstruction requires the reception of specific data packets, belonging to the *base layer*. Then, the reception of additional *enhancement layer* packets improves the quality of the stream. Therefore, a reasonable strategy is to maximize the number of enhancement layer packets subject to the correct reception of base layer packets at each receiver. We show next how to tune MMU to have such a behavior.

TABLE III
BASIC CONFIGURATION OF NITOS NODES

Model	Icarus nodes
CPU	Intel i7-2600 Proc., 8M Cache, at 3.40GHz
RAM	Kingston 4 GB HYPERX BLU DDR3
Storage	Solid State Drive 60GB
WiFi cards	two Atheros 802.11a/b/g/n (MIMO)
OS	3.2.0-31-generic Ubuntu precise
Driver	compat-wireless version 3.6.6-1-snpc

We revisit the example of Fig. 3 and set $x = 1$ so that all links have unit capacities. Next, we tag the packets belonging to the base layer video to distinguish them from the enhancement layer packets. The video stream of each session is modeled by a superposition of two Poisson processes with $\lambda_{\text{base}} = 0.2$ and $\lambda_{\text{enh}} = 0.8$.

Using the penalty approach explained in the previous subsection, it is possible to achieve throughput maximization subject to rate 0.2 at each receiver. We choose $g_u^{(1)}(x) = g_u^{(2)}(x) = x$, $\xi_u^{(1)} = 0.2$ and $\xi_u^{(2)} = 0.2$ for all $u \in U^{(1)}, U^{(2)}$ respectively. However, additionally to achieving a specific throughput rate requirement, we require the reception of specific packets. To cope with this added constraint, we impose a strict priority rule at all transmission queues $Q_l^{(c)}$: *enhancement layer packets are only served if there are no base layer packets left in the queue.*

The resulting experiments for this scenario are shown in Table II. The combination of MMU with the priority rule provides delivery ratio of base layer packets very close to the ideal 100%. The small loss is attributed to randomness of the arrivals. Moreover, when the base layer packet delivery ratio is less than 100%, the enhancement layer counterpart is very small. Conclusively, our policy achieves the high-level goal to combine guaranteed delivery with optimal performance.

V. EXPERIMENTATION IN WIRELESS TESTBED

To demonstrate the practicality of the MMU policy, we develop a prototype implementation in NITOS testbed [17]. NITOS is a heterogeneous outdoor testbed, where two types of networks are used: a wireless network with IEEE 802.11a/b/g/n protocol and a wired network using Gbit Ethernet. Being partly deployed in a building roof, NITOS is a non-RF-isolated wireless testbed. To eliminate interference we employed 802.11a, which is not used by commercial 802.11 products in Greece. The NITOS nodes feature a 3.4GHz Intel i7 processor and two Atheros wireless cards. The main hardware and software specifications of the nodes are depicted in Table III.

A. Implementation Framework

The implementation is based on the Click Modular router framework [18]. Click facilitates experimentation and evaluation of scheduling and flow control algorithms in real systems. It runs as a user-level daemon at each node and via the libpcap library it provides full control on packet transmission. Our implemented framework includes mechanisms for estimating channel quality, forming a queue structure, exchanging queue backlog information, and splitting time into virtual slots.

Estimating Channel Quality. To evaluate channel quality, we adopted the ETT estimation algorithm of Roofnet [19]. Nodes periodically broadcast probes which are used to estimate the successful transmission probability. With this process every node periodically obtains a table with the qualities for each channel rate/neighbor pair. Using this table, the $\mu_l^{\max}$ parameters are determined. This mechanism is known to incur negligible throughput overhead [19], [20].

Queue Structure. We implement the transmission queues $Q_l^{(c)}$ on each node and we create a counter for each $D_l^{(c)}, Z_l^{(c)}$ virtual queue. The counter $Z_l^{(c)}$ may take non-integer values. Each of these internal queues/counters is created upon the arrival of the first packet of a new session. This allows session generation "on the fly". The queues are removed after a period of inactivity.

Exchanging Queue Backlog Information. To compute $W_l^{(c)}$ from (25), each node broadcasts periodically the backlog size of all its transmission queues $Q_l^{(c)}$. If a node u is a receiver for some session c, it broadcasts $Q_l^{(c)} + Y_u^{(c)}$ instead. The broadcast messaging is repeated once every second. Prior experiments suggest that more frequent broadcasts incur visible throughput overhead, while rarer broadcasts may affect the delay performance due to obsolete queue information.

In the proposed schemes, the routing is based on fixed multicast trees. Thus $m_l^{(c)}$ parameters are predefined and known. However, in our implementation, it is possible to use the backlog exchange mechanism to transport information about $m_l^{(c)}$, should these be time-varying.

Virtual Slots. In order to simplify the implementation we use the concept of the virtual slot. Each node keeps an internal timer that expires once every slot. Upon counter expiration the policy selects the next queue to be served and for the duration of the next slot the decision remains fixed. The slot duration is set to 100msecs, equal to 1/10 of the broadcasts period. Small values for the slot duration improve delay and reduce throughput fluctuations but burden the CPU of the device. We leave the investigation of optimum slot duration for future work. We note that the implementation of MMU is not tied to the idea of the virtual slot.

B. Policy Implementation

We modify our proposed policy so that it can operate on a network with wireless channels. Due to interference, some wireless links cannot be activated simultaneously. A well known link activation policy is the *maxweight* policy, proposed in [21] for stabilizing mobile packet networks. Maxweight activates at each slot the set of links that maximize the sum products $\sum_l \mu_l^{\max} W_l^{(c)}(t)$, effectively preferring links with higher capacity. In our setting, the activation of the transmitting nodes is automatically selected by the IEEE 802.11 protocol. It remains to choose the activation of a session and a receiving link, subject to the activated nodes. Using intuition from the maxweight policy we propose the following heuristic.

Maximum Multicast Utility for Wireless (MMU-W) Policy

Parameter Selection, Packet Dropping, Receiver-End Flow Control, Scheduling on Wired Links: same as in MMU.

Scheduling on Wireless Links: Calculate $W_l^{(c)}(t)$ using (25). On a wireless node, choose the link-session pair

$$(l^*, c^*) \in \operatorname{argmax}_{(l,c)} \mu_l^{\max} W_l^{(c)}(t) 1_{[W_l^{(c)}(t)>0]}$$

ties broken arbitrarily. Then, allocate the rate

$$\mu_{l^*}^{(c^*)}(t) = \begin{cases} \mu_{l^*}^{\max} & \text{if } W_{l^*}^{(c^*)}(t) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mu_l^{(c)}(t) = 0$ for all the other link-session pairs.

C. Experiments and Results

We conduct experiments on the specific topology of Figure 5. Five NITOS nodes are used: Alice and Bob are connected via Ethernet while Bob is connected to the other three nodes via wireless. The nodes are configured to run the MMU-W policy. The wireless links use fixed physical rates instead of the 802.11 rate adaptation scheme. In particular we set the physical rates to 18Mb/s, 6Mb/s and 6Mb/s for the links to Carol, Dave, and Erin respectively. The physical rate of the wired connection is 1Gb/s.

We consider two sessions, A and B, each with traffic rate 14Mb/s. The source node for both sessions is Alice and the multicast receivers are shown in Fig. 5. To generate packets we use two UDP streams created with the Iperf tool [22]. We run the Iperf tool on external nodes to avoid polluting the CPU measurements. The receiver rate requirements are 4.5Mb/s for Bob, ξ_{C}Mb/s for Carol, 1.7Mb/s for Dave and ξ_{E}Mb/s for Erin, where the values $\xi_{\mathrm{C}}, \xi_{\mathrm{E}}$ are chosen differently per experiment. The objective is to satisfy all receiver rate requirements as well as maximize throughput.

D. Throughput

We show the measured instantaneous and average throughput for two scenarios. The instantaneous throughput is computed as the average over 1sec periods. In the first scenario we choose $(\xi_{\mathrm{C}}, \xi_{\mathrm{E}}) = (2.8, 1.7)$, see Fig. 6. The objective is achieved because all receiver requirements are satisfied and the excess wireless resource is allocated to the receiver with the highest capacity, i.e. Carol. We observed that the wireless medium was fully utilized. In the second scenario, we reverse the requirements of Carol and Erin, $(\xi_{\mathrm{C}}, \xi_{\mathrm{E}}) = (1.7, 2.8)$, see Fig. 7. The theoretical total throughput is smaller in this case due to Erin's low physical rate and high requirement.

E. CPU Occupancy

Our framework is implemented on user-level click. We observed the user-level CPU occupancy using the vmstat command. The occupancy was 8-10% for the whole duration of experiments, which is encouraging. We note, that a kernel-level implementation can improve this figure further. The CPU usage was the same at all nodes, indicating that our policy does not incur extra burden on the sources. Additionally, it was largely independent of data rates used, which implies

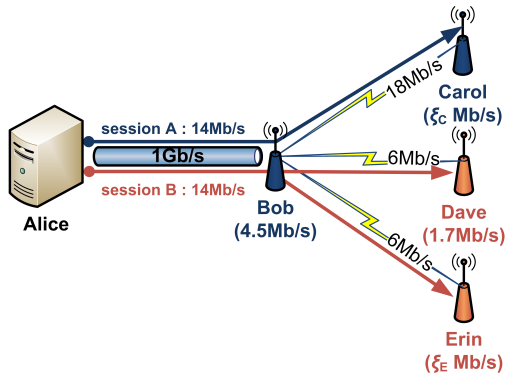

Fig. 5. Experiment topology with five NITOS nodes. Two sessions A and B are generated at Alice, forwarded to Bob via a wired connection, and then distributed to Carol, Dave, and Erin through wireless. The Figure shows the rate requirement per receiver (in parentheses) and the physical rate per link.

that packet operations and queue maintenance have a minor contribution to the CPU occupancy. We plan to present analytic CPU experiments of the policy in future work.

VI. Conclusion

We proposed a distributed control scheme that maximizes utility in multirate multicast. The performance is analyzed and shown to be near-optimal. Several enhancements of the policy are described including a priority rule for base layer packets, and a modification for 802.11 wireless devices. The scheme is implemented in a wireless testbed and its applicability is demonstrated. In future work, we plan to derive the optimal policy for general wireless networks and to experiment further in larger topologies, investigating delay and CPU occupancy.

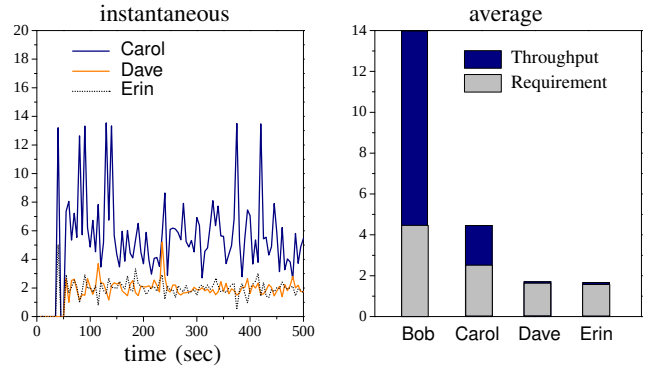

Fig. 6. **Scenario 1:** $(\xi_C, \xi_E) = (2.8, 1.7)$. Instantaneous and average throughput (Mb/s) are shown.

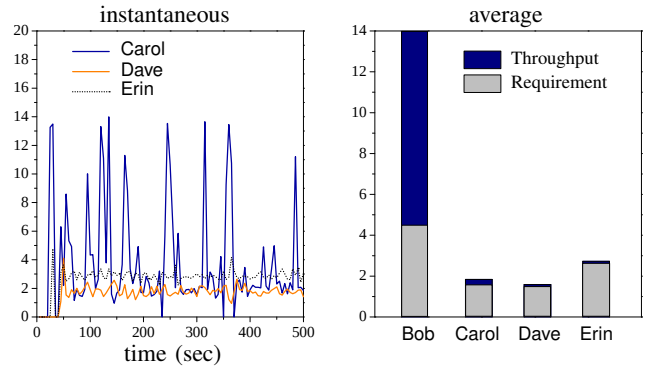

Fig. 7. **Scenario 2:** $(\xi_C, \xi_E) = (1.7, 2.8)$. Instantaneous and average throughput (Mb/s) are shown.

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