Medium Access Games: The Impact of Energy Constraints

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***Abstract*—In this paper we consider random medium access schemes for devices that support sleep modes, i.e. turning off electronic compartments for energy saving. Due to hardware limitations, sleep mode transitions cannot occur at the medium access timescale. Thus, we develop a two level model, consisting of a fast timescale for transmission scheduling and a slower timescale for the sleep mode transitions. We take a game theoretic approach to model the user interactions and show that the energy constraints modify the medium access problem significantly, decreasing the price of anarchy. Our results give valuable insights on the energy–throughput tradeoff for contention based systems.**

I. Introduction

In an attempt to minimize their energy footprint and/or maximize the battery lifetime, existing wireless devices support radio *sleep modes*. A generic wireless terminal consists of several building blocks with the RF transceiver (radio) contributing significantly to the overall energy consumption. Existing wireless devices support radio sleep modes that turn off specific blocks, to minimize their energy consumption while inactive. For example, as shown in Table I the CC2420 transceiver ([1]) provides three different low power modes.

TABLE I
Switching time and energy consumption of a CC2420 radio

Power mode	Switching time(ms)	Switching Energy (μJ)	Current Consumption (μA)
Tx	0	0	10000
idle	0.1	1.035	426
power down	1.2	42.3	40
deep sleep	2.4	85.7	0.02

In the deepest sleep mode, both the oscillator and the voltage regulator are turned off, thus requiring the least amount of current. However, this comes at an increased switching cost in terms of energy and latency. On the other hand, the idle mode provides a quick and energy inexpensive transition back to the active state, but at the cost of higher current. To address this tradeoff, the authors of [2] propose a scheme to dynamically adjust the power mode according to the traffic conditions in the network. For low traffic scenarios deep sleep should be preferred since most of the time the nodes tend to be inactive.

Several energy aware MAC protocols have been proposed, either centralized or distributed ones, to resolve contention. However, most of them rely on the willingness of the nodes to comply with the protocol rules. Hence, they are vulnerable to selfish users that may deviate from the protocol in order to improve their own performance. Thus, the following works use game theory to model the interaction of self–interested entities competing for the common medium. In [3], the Nash Equilibrium Points (NEPs) of a slotted Aloha system of selfish nodes with specific quality-of-service requirements is studied. Generally, selfish behavior in medium access leads to suboptimal performance. In [4] a prisoner's dilemma phenomenon was identified for generalized Aloha protocols.

A decrease in system throughput, especially as the workload increases, due to the selfish behavior of nodes is observed also under energy constraints ([5], [6]). In [7] the problem of minimizing the energy consumption for given throughput demands for a contention MAC is studied. Whenever the demands are feasible, there exist exactly two NEPs and a proper greedy mechanism converges always at the best one.

In this paper we introduce an additional level of decision making capturing the ON-OFF strategy of the terminals over the classic Aloha. Thus, we model contention for the medium as a game, where users with specific energy constraints *select both the proportion of time that they sleep and their medium access probabilities*. To the best of our knowledge, this is the first work that addresses the interplay between contention and energy consumption *for systems that support sleep modes*. In particular,

- we characterize the throughput optimal strategy under energy constraints, which differs from traditional Aloha and serves as a reference point. We also provide a distributed counterpart which focuses on fairness.
- we formulate contention as a non-cooperative game and show that it has a unique NEP and bounded Price of Anarchy (PoA), i.e. contrary to intuition from prior works we find that energy constraints reduce the negative impact of selfish behaviour.
- based on the rationality of the users, we derive a modified strategy, which allows the users to observe competition. This policy is more efficient but has multiple NEPs.

The main characteristics of slotted Aloha are also apparent in most contemporary contention-based systems, such as the IEEE 802.11, where the actual throughput breaks down significantly as the number of users and the message burstiness increase (see [8]). Despite its limited applicability and mainly due to its simplicity, Aloha remains a tractable insightful tool for studying such systems. Consequently, our results provide insights for other contention based systems.

II. System model

We consider a communication scenario of $N = |\mathcal{N}|$ mobile terminals transmitting to a common destination (e.g. uplink to a Base Station). Time is slotted and within each timeslot each user may select either to transmit or to stay silent. Medium access is performed probabilistically, according to

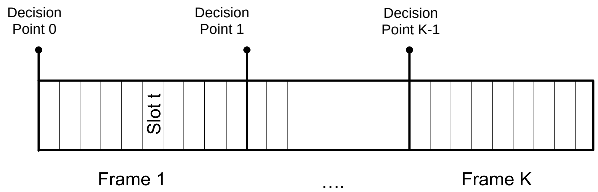

Fig. 1. The structure of a superframe

a slotted Aloha protocol, where a collision occurs whenever two or more terminals transmit concurrently. Each terminal has always packets at its buffer for transmission (i.e. saturated queue), but it has limited energy resources. Each device i is characterized by an energy budget $\tilde{e}_i$, representing either its available battery power or the maximum energy it is willing to expend. In order to save energy it can turn into a sleep mode, where most of the circuits are turned off. For analytical tractability we assume that each terminal may be in one out of two possible operation modes, ON or OFF.

In general, a mode transition incurs significant energy and time (delay) costs. Besides, due to hardware limitations the time required for a mode transition is of the order of msec, much larger than the duration of a timeslot. Consequently, transition at the timeslot level is neither feasible nor desirable. Hence we introduce a new timescale (we call it frame) where the mode switching takes place. Several timeslots constitute a frame. The beginning of each frame is a decision point, where a node may change its operation mode. Within a frame, the nodes keep their mode fixed and may access the medium randomly with probability p. This is also assumed fixed on a per frame basis. For decision reasons and without affecting the operation of the system, we introduce yet another timescale, that of the super frame, where an arbitrary number of frames, say K, forms a superframe (Fig. 1)

A binary vector $\boldsymbol{q}_i(j) = \{0,1\}^K$, represents the ON or OFF state of user i on superframe j, and p_i its access probability (i.e. the access probability is selected only once per superframe). In practice the mobile terminals are autonomous and due to the limited knowledge available node level, synchronization of sleep modes is a difficult task. On the other hand, probabilistic ON-OFF has been deemed a feasible strategy ([9]). Thus, we focus on a probabilistic version of the aforementioned problem. Each user i is characterized by a probability of being ON, denoted with q_i and a medium access probability p_i. In matrix notation the strategy space can be written as $\mathcal{I} = \{\boldsymbol{p}, \boldsymbol{q}\}$, with $\boldsymbol{p} = [p_1, p_2, \dots p_N]$ and $\boldsymbol{q} = [q_1, q_2, \dots q_N]$.

Proposition 1: In our setting, the throughput of a user i can be defined as the number of successfully exploited slots per unit time, and is a random variable with an expected value of:

$$\bar{T}_i(\boldsymbol{p}, \boldsymbol{q}) = p_i q_i \prod_{j \in \mathcal{N} \setminus i} (1 - p_j q_j) \overset{p_i q_i \neq 1}{=} \frac{p_i q_i}{1 - p_i q_i} \prod_{j \in \mathcal{N}} (1 - p_j q_j) \quad (1)$$

Proof: All the proofs are omitted from this extended abstract version. ■

Note that the throughput of user i is an increasing function of p_i and q_i, but decreasing in the number of terminals N contending for the medium. The energy cost of user i is a random variable with a mean value of $\bar{E}_i(p_i, q_i) = q_i(c_1 + c_2 p_i)$, where c_1 is the energy consumption while ON and c_2 the additional cost imposed by the transmission. Obviously, in order to transmit, the node has to be ON. Here, we do not consider the energy consumption of the transition itself.

III. The Impact of Constrained Energy Resources on the System Throughput

First, we would like to find the ON–OFF and the medium access probabilities that maximize the collision–free utilization of the medium and consequently the throughput of the system. This can be formally expressed as the following optimization problem:

$$\begin{aligned} \underset{\mathcal{I}=\{\boldsymbol{p},\boldsymbol{q}\}}{\text{maximize}} \quad & \sum_{i=1}^{N} \bar{T}_i(\boldsymbol{p}, \boldsymbol{q}) \\ \text{s.t.} \quad & \bar{E}_i(p_i, q_i) \le \tilde{e}_i \quad \forall i \\ & \{p_i, q_i\} \in [0,1]^2 \quad \forall i \end{aligned} \quad (2)$$

The intuitive explanation of the energy constraint is the fact that each battery cycle provides $\tilde{e}_i$ resource and thus, the maximum energy constraint is directly mapped to a minimum recharge time for the mobile.

Without loss of generality, we assume that the users are ordered in decreasing energy budget, i.e. $\tilde{e}_1 \ge \tilde{e}_2 \ge \dots \ge \tilde{e}_N$. In the Aloha setting, where no energy constraints exist, the throughput optimal strategy would be the one that eliminates contention, i.e. to force only a single user, say user k, to access the medium with probability $p_k = 1$ in each frame. In our scenario though, due to the energy constraints, users may not be able to stay constantly ON (i.e $q_k = 1$) or transmit with $p_k = 1$. Then, what is the best way to exploit the available energy resources? For each user we need to find the amount of energy to spend for staying ON during the frames and the portion of energy for transmitting.

The centralized Algorithm 1 yields the throughput–optimal probabilistic strategy. The main idea behind this algorithm is that contention may or may not be beneficial, depending on the energy constraints of the users. Namely, an additional user is useful if and only if the energy resources of the already active users are not sufficiently large, leaving thus the medium underutilized. An additional user introduces a gain due to the exploitation of the empty frames, but also a loss, due to the collisions whenever he is concurrently active within a frame with someone else. If the average gain is greater than the induced loss, it is beneficial for the system to be enabled.

Algorithm 1 Optimal probabilistic frame scheduling

Order users in decreasing $\tilde{e}_i$. Without loss of generality, we reassign the indices such that $\tilde{q}_1 \ge \tilde{q}_2 \ge \dots \ge \tilde{q}_N$
$\boldsymbol{q} \leftarrow \boldsymbol{0}, j \leftarrow 1$
while $j \le N$ **and** $\sum_{i=1}^{j} \frac{q_i}{1 - q_i} < 1$ **do**
    $q_j \leftarrow \tilde{q}_j, j \leftarrow j + 1$
end while

The algorithm above promotes the k less energy constrained users ($0 \le k \le N$ depends on actual constraints) and suppresses the rest. It will only serve as a performance benchmark, since it is a centralized algorithm that introduces important coordination and fairness issues.

A. A distributed fair algorithm

In order to capture the notion of proportional fairness we substitute the original objective function with the following: $U(\boldsymbol{p},\boldsymbol{q}) = \sum_{i=1}^{N} w_i \log \bar{T}_i$. The multiplicative factor w_i can be used to balance the throughput among the users of the system at will. For example, the value $w_i = \frac{\tilde{e}_i}{\sum_{k\in\mathcal{N}} \tilde{e}_k}$ would allow us to split the throughput proportionally to the energy budget of the users. By proper reformulation, the objective function can be rewritten as $U(\boldsymbol{p},\boldsymbol{q}) = \sum_{i=1}^{N} \log\left[(p_i q_i)^{w_i}(1-p_i q_i)^{1-w_i}\right]$. This is a separable function that leads to a fully distributed implementation, requiring minimal information exchange. Actually the only information required is the value of the total energy available in the terminals, namely $\sum_{k\in\mathcal{N}} \tilde{e}_k$. The solution of this optimization problem is of the form $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{a}\}$ with $\boldsymbol{a} = [\min\{w_1, \tilde{q}_1\}, \min\{w_2, \tilde{q}_2\}, \ldots, \min\{w_N, \tilde{q}_N\}]$.

B. A modified strategy

Up to now we assumed that each user makes a decision once for his strategy and applies it forever. As a result, user k whenever active, transmits with $p_k = 1$, independently of the number of active users within a frame. Thus, whenever two or more users select to transmit within a frame they receive zero payoff, but consume energy. Based on these, a rational player would be expected to backoff whenever a collision is detected. Although the terminal cannot switch off in a crowded frame, due to the switching time overhead incurred, it may reduce its access probability. This way, it would avoid spending energy on colliding transmissions and could utilize these savings for pursuing further contention–free frames. Building on this idea we propose the following modified strategy.

Any active user attempts a transmission within the first timeslot of the current frame. If the transmission succeeds he selects a medium access probability of $p_i = 1$. Otherwise he adjusts his strategy, and reduces his transmission probability to $\tilde{p}_i$. It can be shown that this strategy always yields better throughput than the original one. The expressions for throughput and energy consumption are respectively given by:

$$\bar{T}_i = q_i \left\{ (1-\tilde{p}_i) \prod_{j\in\mathcal{N}\setminus i} (1-q_j) + \tilde{p}_i \prod_{j\in\mathcal{N}\setminus i} (1-\tilde{p}_j q_j) \right\} \quad (3)$$

$$\bar{E}_i = q_i \left\{ c_1 + c_2 \left[\tilde{p}_i + (1-\tilde{p}_i) \prod_{j\in\mathcal{N}\setminus i} (1-q_j) \right] \right\} \quad (4)$$

Algorithm 2 yields the throughput optimal strategy, by categorizing users into three groups, namely aggressive, conservative and passive ones. The former capture the medium whenever they are active (ON), the second transmit only whenever they sense an empty frame and the last do not participate at all. It can be shown that in the optimal scheduling, there exists at least one aggressive and one conservative user.

Algorithm 2 Modified optimal probabilistic frame scheduling

1: Search over $\mathcal{B} = \{\mathcal{A}, \mathcal{C}, \mathcal{P}\}$, i.e. the set of all the possible partitions of $\mathcal{N}$ of size 3, with $|\mathcal{A}| \geq 1$ and $|\mathcal{C}| \geq 1$ for the throughput optimal assignment:
2: **for all** $i \in \mathcal{A}$ (% *aggressive*) **do**
3: $\quad \{\tilde{p}_i, q_i\} = \{1, \min\left\{\frac{\tilde{e}_i}{c_1+c_2}, 1\right\}\}$
4: **end for**
5: **for all** $k \in \mathcal{C}$ (% *conservative*) **do**
6: $\quad \{\tilde{p}_k, q_k\} = \left\{0, \min\left\{\frac{\tilde{e}_i}{c_1+c_2\prod_{j\in\mathcal{N}\setminus k}(1-q_j)}, 1\right\}\right\}$
7: **end for**
8: **for all** $j \in \mathcal{P}$ (% *passive*) **do**
9: $\quad \{\tilde{p}_j, q_j\} = \{0, 0\}$
10: **end for**

IV. Game Theoretic Model

Previously we derived probabilistic medium access protocols that require coordination of actions of the users involved. However, in an autonomous setting as the one considered here, individuals may not comply with the rules imposed by the protocol. Actually, users may exhibit selfish behaviour and select the strategy that maximizes their own utility, namely their individual throughput, at the expense of others. Thus, we model the user interaction/contention as a non-cooperative game.

A non-cooperative game is defined by a set of players, a set of strategies and a metric that indicates the preferences of the players over the set of strategies. In our case we have:

- **Players**: N users
- **Strategies**: user's i set of feasible medium access and ON–OFF probabilities $\mathcal{I}_i = \{p_i, q_i : \bar{E}_i \leq \tilde{e}_i \text{ and } 0 \leq p_i, q_i \leq 1\}$
- **User preferences** represented by a utility function $U_i(\mathcal{I}_i)$; user i prefers strategy $\hat{\mathcal{I}}_i$ to $\mathcal{I}_i$ iff $U_i(\hat{\mathcal{I}}_i) > U_i(\mathcal{I}_i)$.

For the initial optimization problem (2) the utility function of user i is defined as $U_i(\mathcal{I}_i) = \bar{T}_i = p_i q_i \prod_{j\in\mathcal{N}\setminus i}(1-p_j q_j)$. By using the KKT conditions we derive the following:

Lemma 1: The throughput optimal strategy for user i is $\{p_i, q_i\} = \left\{1, \min\{\frac{\tilde{e}_i}{c_1+c_2}, 1\}\right\}$. The resulting game has a unique NEP, described by the strategy $\mathcal{I}^* = \{\mathbf{1}, \boldsymbol{q}^*\}$, with $q_i^* = \frac{\tilde{e}_i}{c_1+c_2}$.

From the above we may deduce that at the NEP we receive throughput, only when a single user is ON within a frame. Given the NEP of the game we may quantify the performance loss arising due to the selfishness of the individuals, by using so called *Price of Anarchy* (PoA) metric. This is the ratio of the value of the objective function at the global optimum to its value at the NEP and in our setting is given by:

$$\text{PoA} = \frac{\sum_{i\in\mathcal{S}} \frac{\tilde{e}_i}{c_1+c_2} \prod_{j\in\mathcal{S}\setminus i}(1-\frac{\tilde{e}_j}{c_1+c_2})}{\sum_{i\in\mathcal{N}} \frac{\tilde{e}_i}{c_1+c_2} \prod_{j\in\mathcal{N}\setminus i}(1-\frac{\tilde{e}_j}{c_1+c_2})} \geq 1, \quad (5)$$

where $\mathcal{S}$ is the set of enabled users at the global optimum.

Whereas in the classic Aloha games the PoA is unbounded, in our energy constrained Aloha setting the PoA is bounded, since the energy constraints impose a fictitious pricing scheme. The PoA grows unbounded only when at least two users have unconstrained energy resources. Then, these users (e.g. power plugged stations) would involuntarily act as jammers for each other and for all the others yielding hence zero system throughput.

A. The modified strategy as a non-cooperative game of perfect information

Here, we consider the game arising from the modified strategy. In this setting, we derive a non–cooperative game of perfect information, where at each iteration, given the parameters $\alpha = \prod_{j\in\mathcal{N}\setminus i}(1-q_j)$ and $\beta = \prod_{j\in\mathcal{N}\setminus i}(1-\tilde{p}_j q_j)$ a user selects its best response. By *best response* we mean that each user updates his decision variables, so as to maximize its utility function, in response to the others' actions.

Theorem 1: The best response strategy of user i is:

$$\tilde{p}_i = \begin{cases} 1, & \text{if } [c_1 + c_2\alpha]\,\beta > (c_1+c_2)\alpha, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

$$q_i = \frac{\tilde{e}_i}{c_1 + c_2\,[\tilde{p}_i + (1-\tilde{p}_i)\alpha]} \quad (7)$$

The arising modified game has multiple NEPs.

V. Numerical results

We perform some simulations to quantify the throughput performance of the proposed schemes. We assume a network of $N = 5$ terminals with energy constraints given by $\tilde{e} = [30, 25, 15, 10, 5]$ and $\{c_1, c_2\} = \{50, 70\}$ units. We slightly abuse the definition of PoA, by using the modified optimal as the performance benchmark, i.e.

$$\text{Performance\_ratio}_X = \frac{\text{Throughput of modified optimal}}{\text{Throughput of scheme X}} \quad (8)$$

Thus, the figures depict the performance degradation in comparison to the modified optimal. For the modified game we depict the PoA, the price of stability (PoS), defined as the ratio of the throughput at the optimum to the throughput at the best NEP, and the mean performance. Regarding the initial setting, we depict the performance degradation of the original optimal, the fair and the initial game theoretic scheme.

Initially, we consider how the energy constraint of the less energy constrained user affects the performance of the system as a whole. As shown in the first figure, the additional power budget increases the performance degradation due to the additional collisions caused. The system stabilizes for $\tilde{e}_1 = c_1 + c_2$, where user 1 has sufficient energy to capture the whole medium on his own. Next, we depict the impact of the transmission cost c_2 on the performance. In a scenario of low energy constraints the increased transmission cost makes the users less aggressive, leading thus to reduced collisions. Both figures indicate that the modified strategy of backing off when a collision is detected may dramatically improve performance.

VI. Conclusion

This work is a first step towards characterizing the energy–throughput tradeoff for mobile devices that support sleep

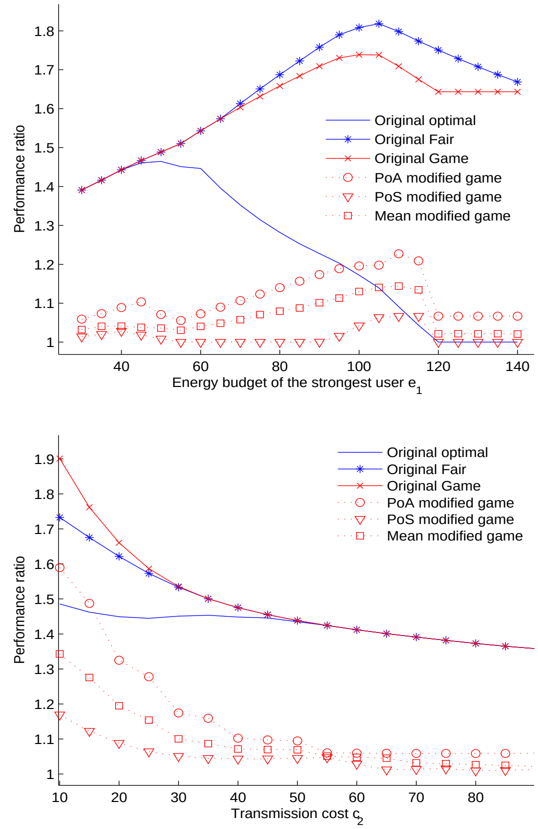

Fig. 2. The throughput degradation compared to the optimal

modes and operate according to contention medium access schemes. We showed that energy constraints may reduce contention and lead to better exploitation of the medium. Here, we have assumed non–cooperative games of perfect information. The scenario where each involved entity has only a subjective belief on its opponents' strategies is an interesting topic of future study.

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