Extending the percolation threshold using power control

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***Abstract*—In this paper we underline the importance of utilizing unequal powers in wireless ad hoc networks. Recent results from percolation theory indicate that a threshold exists after which a very large randomly positioned ad hoc network becomes disconnected almost surely for a given communication configuration. In this paper we prove that it is possible to extend the region of connectivity by allocating the transmit power of each node in an intelligent manner. We actually show that this is possible even in the case of reducing only the powers where appropriate.**

I. Introduction

The impact of power control in wireless communications is well studied, [1], [2]. In CDMA networks for example, power control is used to minimize power consumption, reduce interference and improve the user perceived quality. In wireless ad hoc networks, power control and topology control is also proposed to improve connectivity, network lifetime and capacity. Usually, the joint optimization of all interested utilities is mathematically intractable but insights are given in partial problems.

On the other hand, there are worries expressed on how the power control can help a distributed wireless ad hoc network. Despite the fact that an efficient distributed power control algorithm is known since 1993 (see [3] and [4]), which has also been extended in many aspects, the work of [5] and [6] demonstrates network regimes where power control becomes inefficient. Up to date, it remains obscured whether power control is useful in a very large wireless ad hoc network.

In this paper we are interested to investigate the effect of power control on connectivity of a very large ad hoc network. A well-reputed tool for studying connectivity on infinite networks is percolation theory. When a network with random topology percolates, there exists a unique, infinite in size, connected component of nodes [7]. This implies that given both sender and recipient are part of this component (which happens for each one with probability θ_p called percolation probability), they can communicate being arbitrarily far away the one from the other. This study becomes more interesting when applied to more realistic models that capture interference by use of signal to interference plus noise (SINR) measure, as in [8].

Dousse at al., in [8], using the notion of orthogonality factor γ, proved that a network using the realistic SINR model percolates for fixed power allocation and a certain range of parameters. More precisely, they show that a threshold $\gamma_c > 0$ exists that separates the supercritical area (area where $\theta_p > 0$) from the subcritical area (where $\theta_p = 0$). In this paper, we show that there exist a power configuration which can be found using a distributed power control method (e.g. [4]), that respects average or maximum power constraints, and has a strictly larger supercritical area (i.e. $\gamma_{PC} > \gamma_c$). This equivalently means that by using the same power resource in an intelligent manner, it is possible to reinforce the network topology so it can withstand higher levels of interference. It also implies that transmitting with equal powers in a random network is always connectivity-wise suboptimal.

In the first result, we show that using the distributed power control algorithm, a gain in average power is obtained. Then the nodes can normalize their powers using the average gain and the extra amount of power is used to mitigate an increase of interference factor. Using this approach, the maximum power constraint is however violated. In the second result, we repeat the same process but this time we allow nodes to increase the powers up to the maximum constraint. We show that even in this case, where the nodes are actually allowed only to reduce their powers in comparison to the constant power case, there is still a strictly positive gain of tolerated interference. Another interesting question, arising from simulations, is whether this gain can be improved by allowing topology changes.

The paper is organized as follows. In section II the communication model is described and several features are discussed. In section III the methodology of connectivity reinforcement is explained and in section IV it is extended to the case of maximum power constraint. In section V, interesting simulation results are showcased. The paper is finally concluded in section VI.

II. Communication model

The wireless network consists of a countably infinite set of nodes (vertices) $\mathcal{V} = \{v_1, v_2, \dots\}$ with each node positioned on the plane according to a two dimensional Poisson spatial process with some density λ. We apply the marking of power, having element v_i bearing the power p_i and define $\mathcal{E}$ the set of bidirectional links (edges) with elements all unordered pairs $\{v_i,v_j\}: v_i,v_j \in \mathcal{V}$ such that the following inequalities hold

$$SINR_{ij} = \frac{p_i l_{ij}}{\gamma \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell l_{\ell j} + N_0} \geq \beta \tag{1}$$

$$SINR_{ji} = \frac{p_j l_{ji}}{\gamma \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell l_{\ell i} + N_0} \geq \beta, \tag{2}$$

where γ is an interference coupling multiplicative factor (orthogonality factor), β is the SINR requirements and N_0 is the background noise level. Matrix $\mathbf{L} = \{l_{ij}\}$ contains the attenuation values l_{ij} of the directed path (v_i,v_j), where $l:\mathbb{R}^+ \to \mathbb{R}^+$ is the so-called attenuation function defined here exactly the same way with [8]. That is, for a pair of nodes (v_i,v_j), with positions $X_i,\ X_j$, we write $l_{ij} \doteq l(|X_i - X_j|)$ and

1) $l(x)$ is continuous and, as long as it does not vanish, strictly decreasing.
2) $l(x) \leq 1$, there is no gain in power when the nodes are very close to each other.
3) $l(0) \geq \frac{\beta N_0}{p_{\max}}$, otherwise there would be no links.
4) $\int_0^\infty x l(x) < +\infty$ (converges).

We assume that the nodes have limited available power which translates to $0 \leq p_i \leq p_{\max}, \forall v_i \in \mathcal{V}$. We call $\mathbf{p}^{\mathrm{c}} = p_{\max}[1 \ \ 1 \ \ \ldots \ \ 1 \ \ \ldots]$ the *constant power vector*. Note that under the constant power vector, all nodes transmit at maximum power.

It is evident from the above that every realization of the spatial process yields a graph (network) that depends also on vector $\mathbf{p}$. In particular, $\mathcal{G}(\mathbf{p}) = \{\mathcal{V}, \mathcal{E}(\mathbf{p})\}$ with $\mathcal{E}$ depending on $\mathbf{p}$.

The above described model is called the STIRG model (in [8]) and reduces to the Boolean model (see [7]) if we set $\gamma = 0$.

A. Percolation

Under the STIRG model, as explained in [8], a strictly positive γ_c exists such that using constant power vector, the arising graph $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$ percolates almost surely for any $\gamma < \gamma_c$ and becomes disconnected almost surely for any $\gamma > \gamma_c$. In other words, γ_c is the percolation threshold. The proof is valid for attenuation factor $\alpha > 2$ (see [8]) and density $\lambda \geq \lambda_c$ where λ_c is the percolation threshold for the Boolean model. In this paper, we are interested to keep this property, and we will do so by using supergraphs of $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$.

A question that arises is whether the supercritical area (the one where $\gamma < \gamma_c$ holds), that by [8] provably exists, can be extended by rearranging the powers. Is it possible to do so by using a power control algorithm? Moreover, is it possible to achieve this only by reducing individual powers?

B. Power control model

The goal of power control is to find the minimum power vector that satisfies all SINR requirements (that is β in our case) for all given links. Each node has a weakest link and the above can be reduced to require that all weakest links are satisfied.

Definition 1. Given $\beta > 0$ and a set of links $\mathcal{E}$, we say that $\mathbf{p} \geq 0$ is a valid power vector if (and only if)

$$\forall \{v_i,v_j\} \in \mathcal{E} \quad SINR_{ij}(\mathbf{p}) \geq \beta \Leftrightarrow \min_{(v_i,v_j)\in\mathcal{E}} \frac{SINR_{ij}(\mathbf{p})}{\beta} \geq 1 \tag{3}$$

If there is a valid power vector, then the problem $(\mathcal{E},\beta)$ is said to be feasible.

In 1993, [3], an iterative distributed power control algorithm was introduced that can be applied on an arbitrary given topology and given that the problem is feasible, it yields the optimal vector $\mathbf{p}^{\mathrm{PC}}$ that minimizes all norms. Later, in [4], it was proven that this algorithm, applied on a feasible problem, has a unique fixed point whenever the so called *standard interference function* is used. Any standard interference function should satisfy three requirements, for any vector $\mathbf{p}$ and index i,

- positivity, $I_i(\mathbf{p}) > 0$,
- scalability, $I_i(\mu\mathbf{p}) < \mu I_i(\mathbf{p}) \quad \forall \mu > 1$,
- monotonicity, $I_i(\mathbf{p}) \leq I_i(\mathbf{p}')$ whenever $\mathbf{p}' \geq \mathbf{p}$.

The above axioms imply continuity for the interference function and consequently for the SINR.

One example of a standard interference function is a function that computes the necessary power level so the SINR requirements of the weakest link of each node are satisfied with equality. For our model we could write

$$p_i(k+1) = I_i(\mathbf{p}(k)) = \frac{\beta\gamma \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell(k) l_{\ell i} + \beta N_0}{l_{ij}} \tag{4}$$

where $\{v_i,v_j\}$ is the weakest link of v_i in this case. For each node with zero degree we set $p_i(k) = 0$ for $k \geq 1$. The above function yields the power level of node i at iteration $k+1$ given the power vector at iteration k. This function fulfils all the above axioms and therefore belongs to the category of standard interference functions. As such, by applying this iterative mapping, and given that a valid power vector exists, the optimal power vector $\mathbf{p}^{\mathrm{PC}}$ can be obtained. For any valid power vector $\mathbf{p}$ other than the optimal one, it holds $\mathbf{p}^{\mathrm{PC}} \leq \mathbf{p}$ (element-wise).

In our case, we use as link requirements the set of links of $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$ and therefore the problem is feasible (i.e. the set of valid vectors is nonempty) since $\mathbf{p}^{\mathrm{c}}$ is a valid vector. From the above it is obvious that $\mathbf{p}^{\mathrm{PC}}$ yields a network which is a supergraph of the network obtained by the constant power vector, $\mathcal{G}(\mathbf{p}^{\mathrm{PC}}) \supseteq \mathcal{G}(\mathbf{p}^{\mathrm{c}})$, since all links of $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$ are preserved and a few extra might be added.

C. Power control gain

Here we define the gain of the iterative distributed power control algorithm explained in the previous subsection. The gain at k^{th} iteration can be defined in terms of average power or maximum power. For average power we define the average gain

$$g_{\text{av}}(k) = \frac{p_{\max} - \mathbb{E}[p_i(k)]}{p_{\max}} \tag{5}$$

where the expectation is taken over all elements of $\mathbf{p}$. Similarly, we define maximum gain

$$g_{\max}(k) = \frac{p_{\max} - \max\{\mathbf{p}(k)\}}{p_{\max}} \tag{6}$$

Since the iterative algorithm is proven to be a contraction mapping, it is trivial to show that $g_{\mathrm{av}}(k+1) \geq g_{\mathrm{av}}(k)$ and $g_{\max}(k+1) \geq g_{\max}(k)$ for any $k \in \mathbb{N}$.

D. Interference Graph

Under the STIRG model, for any pair of nodes $\{v_i, v_j\} \in \mathcal{E}$ (hereinafter called 1–hop neighbors) there is a maximum Euclidean distance between those two $d_{\max}^{(1)} = l^{-1}\left(\frac{\beta N_0}{p_{\max}}\right)$. This can be extended in case of k–hop neighbors as $d_{\max}^{(k)} = \sup\{d^{(k)}\} = k d_{\max}^{(1)}$.

Definition 2. $\mathcal{J}(\mathcal{G}(\mathbf{p}), \varepsilon) = (\mathcal{V}_{\mathcal{J}}, \mathcal{E}_{\mathcal{J}})$ is called the ε–interference graph of $\mathcal{G}(\mathbf{p})$ and defined as $\mathcal{V}_{\mathcal{J}} = \mathcal{V}$ and $\mathcal{E}_{\mathcal{J}} = \{\{v_i, v_j\} : \frac{l_{jm}}{l_{im}} > \varepsilon \;\; \forall v_m$ such that $\{v_i, v_m\} \in \mathcal{E}$, and $\frac{l_{in}}{l_{jn}} > \varepsilon \;\; \forall v_n$ such that $\{v_j, v_n\} \in \mathcal{E}\}$.

Assume $\mathcal{G}_R(\mathcal{V}_R, \mathcal{E}_R)$ a reduced version of $\mathcal{G}$ such that the set $\mathcal{V}_R$ contains all elements of $\mathcal{V}$ with the property *part of the infinite component* and $\mathcal{E}_R$ has all unordered pairs of $\mathcal{V}_R$ that are part of $\mathcal{E}$. Note that $\mathcal{G}_R$ is connected whenever $\mathcal{G}$ percolates and an empty set if $\mathcal{G}$ does not percolate.

Lemma 1. *Given that $\mathcal{G}(\mathbf{p})$ percolates, there exist $\varepsilon > 0$ such that the interference graph $\mathcal{J}(\mathcal{G}_R(\mathbf{p}), \varepsilon) = (\mathcal{V}_{R\mathcal{J}}, \mathcal{E}_{R\mathcal{J}})$ is connected (or equivalently; for any pair of nodes $\{v_i, v_j\} \in \mathcal{E}_{R\mathcal{J}}$ there exist a path in $\mathcal{J}$ connecting these pair of nodes).*

Proof: Since $\mathcal{G}(\mathbf{p})$ percolates, we know that $\mathcal{G}_R(\mathbf{p})$ is connected. Since $\mathcal{V}_{R\mathcal{J}} = \mathcal{V}_R$, it is enough to show that for any link of $\mathcal{G}_R(\mathbf{p})$, say $\{v_i, v_j\}$, there exists a path in $\mathcal{J}(\mathcal{G}_R(\mathbf{p}), \varepsilon)$ connecting these two nodes.

Pick a node $v_\ell \in \mathcal{V}_{R\mathcal{J}}$ as a 3–hop neighbor of v_i. Then v_ℓ is k–hop neighbor of v_j, with $2 \leq k \leq 4$. Pick also $\varepsilon = l(4d_{\max})$. Then using properties of the attenuation function, for any 1–hop neighbor of v_i, v_m we get

$$\frac{l_{\ell m}}{l_{im}} \geq l_{\ell m} > l(4d_{\max}) = \varepsilon \tag{7}$$

Similarly, $\frac{l_{\ell n}}{l_{jn}} > \varepsilon$, for any 1–hop neighbor of v_j, v_n. This implies that the path $\{v_i, v_\ell, v_j\}$ exists in $\mathcal{J}(\mathcal{G}_R(\mathbf{p}), \varepsilon)$. ■

E. Network irreducibility

Definition 3. A network (graph) $\mathcal{G}(\mathbf{p})$ is called ε–irreducible if and only if the corresponding ε–interference graph $\mathcal{J}(\mathcal{G}(\mathbf{p}), \varepsilon)$ is connected.

We call a network irreducible whenever there exist positive ε such as the network is ε–irreducible. Using lemma 1, we are in position to observe that, under the STIRG model, whenever network $\mathcal{G}$ percolates the reduced network $\mathcal{G}_R$ is irreducible.

III. Improving the percolation threshold

Using the above definitions we would like to show that the percolation threshold can be improved by using the iterative power control algorithm of (4). We will do so by finding a vector $\mathbf{p}'$ such as for small positive ξ, $\mathcal{G}(\mathbf{p}^{\mathrm{c}}, \gamma_c)$ percolates, $\mathcal{G}(\mathbf{p}^{\mathrm{c}}, \gamma_c + \xi)$ does not percolate and $\mathcal{G}(\mathbf{p}', \gamma_c + \xi)$ percolates. It is also of importance to impose certain constraints on vector $\mathbf{p}'$.

Let us start with vector $\mathbf{p}^{\mathrm{c}}$ and obtain the network $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$. We know that for any $\gamma \leq \gamma_c$ the network percolates. We also know that we can pick any arbitrarily small but positive ξ such as the network will not have an infinite component for $\gamma = \gamma_c + \xi$. Now we apply the iterative algorithm of (4) on $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$ and obtain the vector $\mathbf{p}^{\mathrm{PC}}$. If we define the event $A_M = \{$An arbitrary node has at least one link under the model $M\}$, for the average gain of this procedure we can show

$$\begin{aligned} g_{\mathrm{av}}(\infty) > g_{\mathrm{av}}(1) = \\ &= \frac{p_{\max} - \mathbb{E}\left[p_i^{\mathrm{PC}}(1)\right]}{p_{\max}} > \\ &> 1 - \mathbb{E}[\mathbb{1}\{A_{\mathrm{STIRG}}\}] \geq \\ &\geq 1 - \mathbb{E}[\mathbb{1}\{A_{\mathrm{Boolean}}\}] = \\ &= e^{-\pi\lambda\left(l^{-1}\left(\frac{\beta N_0}{p_{\max}}\right)\right)^2} \end{aligned} \tag{8}$$

Here we have used the fact that a proportion of nodes has no links under the Boolean model, and therefore under the STIRG model as well, and these nodes will set their powers to zero. Inequality (8) implies that the gain in average power will be bounded away from zero by a constant whenever density λ is finite and the noise level N_0 is strictly positive.

Next we scale the power vector by $a_{\mathrm{av}} = \frac{1}{1-g_{\mathrm{av}}(\infty)}$. From the previous it is obvious that $a_{\mathrm{av}} > 1$. Recall that $\mathcal{G}(\mathbf{p}^{\mathrm{PC}}) \supseteq \mathcal{G}(\mathbf{p}^{\mathrm{c}})$ and now $\mathcal{G}(a_{\mathrm{av}}\mathbf{p}^{\mathrm{PC}}) \supseteq \mathcal{G}(\mathbf{p}^{\mathrm{PC}})$ since scaling the powers by a_{av} is equivalent to scale the noise by $\frac{1}{a_{\mathrm{av}}}$ which in turn only improves all links whenever $a_{\mathrm{av}} > 1$. Thus $\mathcal{G}(a_{\mathrm{av}}\mathbf{p}^{\mathrm{PC}}, \gamma_c)$ percolates.

Furthermore, we can show that for any link $\{v_i, v_j\}$ in $\mathcal{G}(\mathbf{p}^{\mathrm{c}})$

$$\begin{aligned} p_i l_{ij} \geq \beta\left(\gamma_c \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell l_{\ell i} + N_0\right) = \\ = \beta\left((\gamma_c + \xi) \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell l_{\ell i} + \frac{N_0}{a_{\mathrm{av}}}\right) + \\ + g_{\mathrm{av}}(\infty) N_0 - \xi \frac{p_i l_{ij} - \beta N_0}{\beta\gamma} > \\ > \beta\left((\gamma_c + \xi) \sum_{v_\ell \in \mathcal{V}\setminus\{v_i,v_j\}} p_\ell l_{\ell i} + \frac{N_0}{a_{\mathrm{av}}}\right) \end{aligned} \tag{9}$$

where the last inequality holds if we pick $\xi < \dfrac{\beta\gamma_c g_{\mathrm{av}}(\infty) N_0}{p_{\max} - \beta N_0}$. This proves that all links in $\mathcal{G}(\mathbf{p}_c, \gamma_c)$ are retained in $\mathcal{G}(a_{\mathrm{av}}\mathbf{p}^{\mathrm{PC}}, \gamma_c + \xi)$ and thus $\mathcal{G}(a\mathbf{p}^{\mathrm{PC}}, \gamma_c + \xi)$ percolates. This new area is strictly larger from the previous whenever $g_{\text{av}}(\infty)N_0 > 0$.

So far we have shown that by choosing $\mathbf{p}' = a_{\text{av}}\mathbf{p}^{\text{PC}}$, where $a_{\text{av}} = \frac{1}{1-g_{\text{av}}(\infty)}$, we can achieve a strictly larger supercritical area. It would be interesting to show something similar for the case when the maximum power constraint is not violated, i.e. for the case when the rescaling of powers is done so the new maximum power is equal to the original one ($p_{\max}$). This is the goal of the following section.

IV. Extension to the improvement

Now we are interested to extend the result of the previous section to the case where the maximum constraint is not violated.

Proposition 1. *The optimal power vector $\mathbf{p}^{\text{PC}}$ of an irreducible network $\mathcal{G}_R(\mathbf{p}^{\text{c}})$ has the property $\mathbf{p}^{\text{PC}} < \mathbf{p}$ (element-wise), for any feasible vector $\mathbf{p}$ other than the optimal.*

Proof: For the optimal vector we know from [4] that $\mathbf{p}^{\text{PC}} \leq \mathbf{p}$. We are interested to show strict inequality. Suppose the there exists index i such that $v_i \in \mathcal{V}_R$ and $p_i^{\text{PC}} = p_i$. Then this implies

$$\begin{aligned}
&I_i(\mathbf{p}) - I_i(\mathbf{p}^{\text{PC}}) = 0 \Leftrightarrow \\
&\gamma \sum_{v_\ell \in \mathcal{V}_R \setminus \{v_i, v_j\}} \frac{l_{\ell j}}{l_{ij}}(p_i - p_i^{\text{PC}}) = 0 \Leftrightarrow \\
&\sum_{v_\ell \in \mathcal{N}_i} \frac{l_{\ell j}}{l_{ij}}(p_i - p_i^{\text{PC}}) + \sum_{v_\ell \in \mathcal{V}_R \setminus \mathcal{N}_i} \frac{l_{\ell j}}{l_{ij}}(p_i - p_i^{\text{PC}}) = 0
\end{aligned} \tag{10}$$

where $\mathcal{N}_i$ is the set of nodes that are 1–hop neighbors of v_i in $\mathcal{J}(\mathcal{G}_R(\mathbf{p}), \varepsilon)$. Since the network is irreducible, there exists ε such that $\mathcal{N}_i \neq \varnothing$. Therefore (10) implies that $p_\ell = p_\ell^{\text{PC}}$ for all $v_\ell \in \mathcal{N}_i$.

Picking $j : v_j \in \mathcal{N}_i$, Repeating the above we can state that $p_\ell = p_\ell^{\text{PC}}$ for all $v_\ell \in \mathcal{N}_j$.

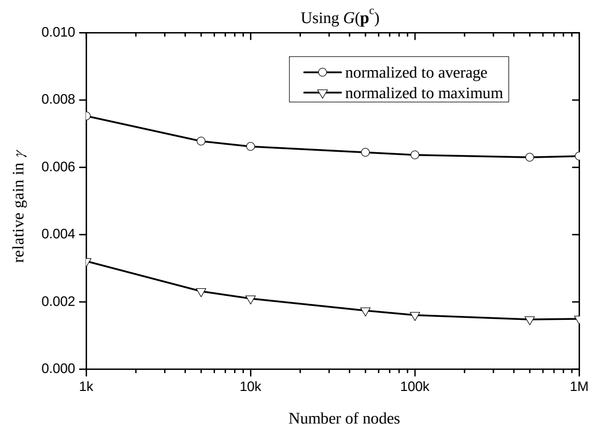
By lemma 1, $\mathcal{J}(\mathcal{G}_R(\mathbf{p}), \varepsilon)$ is connected and there exists a path connecting any pair of nodes belonging to $\mathcal{V}_R$. This results in $p_\ell = p_\ell^{\text{PC}}$ for all $v_\ell \in \mathcal{V}_R$, and we get $\mathbf{p}^{\text{PC}} = \mathbf{p}$ which contradicts the uniqueness of $\mathbf{p}^{\text{PC}}$.

■

From the above it follows that if the constant power vector is not the power optimal vector then $\max\{\mathbf{p}^{\text{PC}}\} < \max\{\mathbf{p}^{\text{c}}\} = p_{\max}$. Since the constant power vector in an infinite randomly positioned network will be power optimal with probability zero, using proposition 1, and the analysis of the previous section, we extract the conclusion that there exist a power vector $\mathbf{p}' = a_{\max}\mathbf{p}^{\text{PC}}$, with $a_{\max} = \frac{1}{1-g_{\max}(\infty)} > 1$, such as $\mathbf{p}' \leq \mathbf{p}^{\text{c}}$ (element-wise), for which the percolation area is strictly larger than that of $\mathcal{G}(\mathbf{p}^{\text{c}})$. Note that the above does not guarantee that the gain in this case is bounded away from zero by a constant.

V. Simulation Results

Simulations can provide useful insights. In the first experiment, figure 1, we were interested to capture the power control gain scaling with the number of nodes in the network. From the figures, there is no definite conclusion to be made, nevertheless the gain does not seem to vanish considering all cases. Particularly, we present two figures, one for the power control case where the graph is fixed to $\mathcal{G}(\mathbf{p}^{\text{c}})$ and one for the case where possible new links are allowed to be added (i.e. using $\mathcal{G}(a\mathbf{p}^{\text{PC}}(5))$). In the second case it seems that the graph remains percolated even when formerly critical links are lost. This implies that topology control can also improve connectivity. In each figure there are two cases presented, one where the powers of all nodes are scaled so they have average power equal to constant power vector (i.e. $\mathbb{E}[\mathbf{p}'] = p_{\max}$) and the second where the powers are scaled without violating the maximum power (i.e. $\max\{\mathbf{p}'\} = p_{\max}$). In the second case we can actually have only decrease in powers of all nodes. The figures show the average relative gain in orthogonality factor after 5 iterations, $g_{\text{av}}(5)$ and $g_{\max}(5)$.



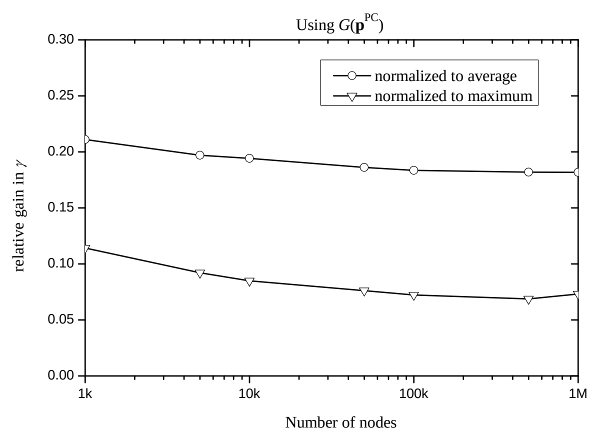


Fig. 1. Average relative gain of γ for the case of unequal powers (top) and unequal power combined with topology changes (bottom), using scaling to average and maximum power.

In the second scenario, figure 2, we are interested to monitor the average and maximum operators on vector $\mathbf{p}^{\text{PC}}$ scaled with number of iterations of the distributed power control algorithm. Note that both measures become smaller than $p_{\max}$

(the figure is equalized with $p_{\max}$). However, the maximum operator has a much slower decrease which implies that for scaling with respect to maximum power, a large number of iterations is required.

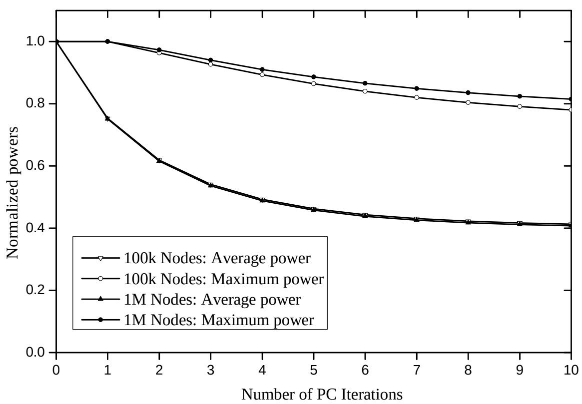


Fig. 2. Average and maximum power of a network with 100k and 1M nodes are shown after several power control iterations.

In the last scenario, figure 3, we showcase the supercritical area improvement for a specific setting. Number of iterations is 20, $p_{\max} = 1$, $N_0 = 0.1$, $l(x) = \min(1, x^{-3})$ and 100k nodes were used.

VI. Conclusion and Future work

We show that the iterative distributed power control algorithm applied to arbitrarily large random networks yields gains for average and maximum power. This result mapped on recent percolation theory results, show that the constant power vector is always suboptimal in terms of connectivity. Using a distributed power control one can minimize the used power while in the same time the connectivity becomes more robust. In order to acquire a more substantial improvement, it is proposed that changes in topology are allowed. As future work, we also propose the investigation of time variant power vectors in order to model fading scenarios.

VII. Acknowledgement

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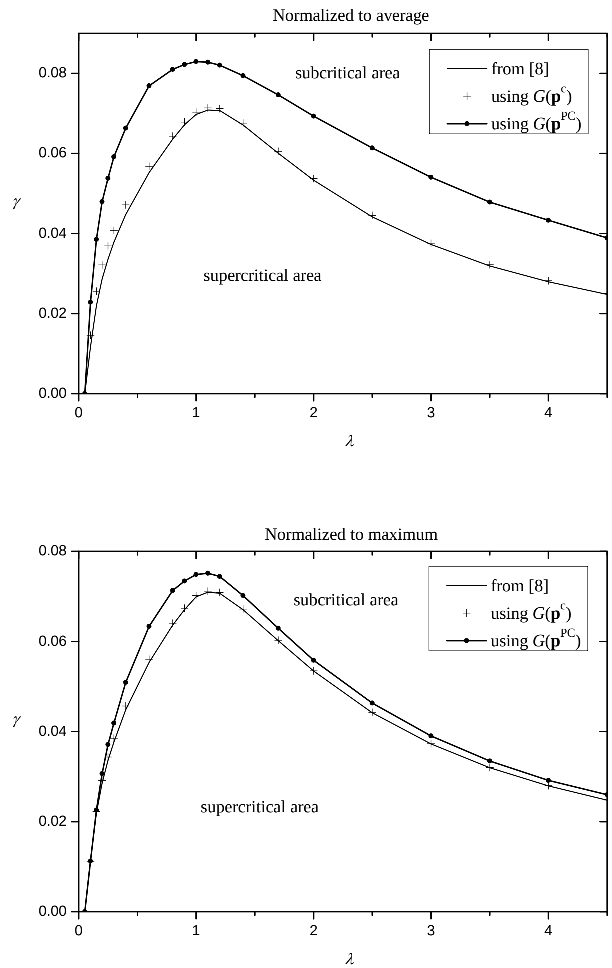

Fig. 3. Percolation threshold for several values of γ and λ. Several cases are shown, using average or maximum rescaling and allowing or not the addition of new links.

References

[1] N. Bambos, “Toward Power-Sensitive Network Architectures in Wireless Communications: Concepts, Issues and Design Aspects”, IEEE Personal Communications Magazine, Vol. 5, pp. 50-59, June 1998.

[2] S. Stanczak, M. Wiczanowski and H. Boche, “Fundamentals of Resource Allocation in Wireless Networks”, Springer, Berlin, Foundation in Signal Processing, Communications and Networking series, 2008.

[3] G. J. Foschini and Z. Miljanic, “A simple distributed autonomous power control algorithm and its convergence”, IEEE Transaction on Vehicular Technology, Vol. 42, No. 4, pp. 641-646, November 1993.

[4] R. Yates, “A framework for uplink power control in cellular radio systems”, IEEE Journal on Selected Areas in Communications, vol.13, pp. 1341-1347, September 1995.

[5] B. Radunovic, J.-Y. Le Boudec, “Power Control is not required for Wireless Networks in the Linear Regime”, proceedings of the sixth IEEE International Symposium on world of Wireless Mobile and Multimedia Networks, p. 417-427, June 13-16, 2005.

[6] V. Kawadia and P. R. Kumar, “Principles and protocols for Power Control in Wireless ad hoc Networks”, IEEE Journal on Selected Areas in Communications, vol.23, pp. 76-88, January 2005.

[7] R. Meester and R. Roy, “Continuum Percolation”, Cambridge University Press, 1996.

[8] O. Dousse, M. Franceschetti, N. Macris, R. Meester and P. Thiran, “Percolation in the signal to interference ratio graph”, Journal of Applied Probability, Vol. 43, Nr. 2, 2006.